Dynamic control of manipulators in operational spaces

KHATIB, O.
Artificial Intelligence Laboratory
Computer Science Department
Stanford University
Stanford, California 94305
U.S.A.

SYNOPSIS Research in manipulator control has focused on the development and application of dynamic models of manipulators. Typical models relate joint variables to generalized torques and by necessity force the resulting control scheme to have two levels. The first level requires coordinate transformations to convert the description of a desired path from Cartesian to joint space. The second level makes use of the arm's dynamic model to calculate generalized torque commands. This first stage of control, the transformation from a Cartesian description of the path into joint trajectories, is very time consuming and prone to problems at kinematic singularities. We propose a new control scheme which is based on the construction of a dynamic model of a manipulator in operational space (task space) rather than joint space. This allows a simple force transformation to replace the difficult conversion of the Cartesian path into joint coordinates. A fundamental advantage of this approach is that the dynamic behavior of the system is controlled in the same space as the path's description, allowing an exact statement of error dynamics in Cartesian space. The system leads to a reduction in the amount of computation and avoids singularity problems. It is also a powerful method to control redundant manipulators, and is the basis for a unique obstacle avoidance scheme. This method has been used to control a PUMA 600 at Stanford University.

1. Introduction

Conventional manipulator control, providing linear feedback compensation to control joint positions independently, is unable to meet the high accuracy and performance required in precision manipulator tasks. Addressing this problem, research has been directed at developing and modelling the dynamic equations of joint motion. Based on these models, the dynamic control problem has been formulated in terms of controlling the behaviour of the manipulator in joint space. However, manipulator action is essentially characterized by the variation, as much in space as in time, of its end effector position and orientation, and the forces and torques it exerts on the environment. The end effector is, in truth, the part of the manipulator most closely linked to the task. In manipulator control, the predominant concern is that the end effector motion and the exerted forces and torques respond accurately to the desired task. At the level of joint motions, concern is limited to issues of the global stability of the articulated mechanism and the satisfaction of the constraints under which it must operate. Approachs to end effector motion control [1,2,3] require task transformation and dynamic compensation, and this leads to high computational complexity. End effector dynamic modeling and real-time control in the presence of obstacles and constraints constitute the central objective of the work [4] presented in this paper.

2. Mathematical Models

The end effector configuration is represented by m parameters describing its position and orientation in a frame of reference R_0. These m parameters will be called *End Effector Configuration Parameters* and represented by x. The geometric and kinematic models of a manipulator are:

$$x = G(q) \tag{1}$$
$$\dot{x} = J(q)\,\dot{q} \tag{2}$$

where q is the vector of the n joint coordinates, and $J(q)$ the Jacobian matrix. For different choices of representation of x, different Jacobians can be defined. An important particular Jacobian, termed the *basic Jacobian*, is defined independently from the parameters x:

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = J_0(q)\dot{q} \tag{3}$$

where v and ω are respectively the linear and the angular velocity vectors *w.r.t.* R_0.

The dynamic model can be written in the form:

$$A(q)\ddot{q} + b(q,\dot{q}) - g(q) = \Gamma \tag{4}$$

with:

$$\begin{aligned} b(q,\dot{q}) &= B(q)[\dot{q}\dot{q}] + C(q)[\dot{q}^2] \\ [\dot{q}\dot{q}] &= [\dot{q}_1\dot{q}_2\ \dot{q}_1\dot{q}_3 \cdots \dot{q}_{n-1}\ \dot{q}_n]^T \\ [\dot{q}^2] &= [\dot{q}_1^2\ \dot{q}_2^2 \cdots \dot{q}_n^2]^T \end{aligned} \tag{5}$$

where $A(q)$, $B(q)$ and $C(q)$ are respectively the $n \times n$ kinetic energy matrix, the $n \times n(n-1)/2$ matrix of Coriolis forces, and the $n \times n$ matrix of centrifugal forces; $g(q)$ is the gravity forces column matrix.

3. Operational Coordinates

Definition: An *Operational Coordinate System* is a coordinate system formed by an *independent* set of m_0 end effector configuration parameters.

Cartesian coordinates (x, y, z) are the most common representation for end effector position, whereas several different representations have been used for the orientation parameters. All orientation descriptions, other than those using angular parameters, are redundant. Euler angle representations of the rotation, and more generally *any* minimal representation, will be singular in certain configurations. This type of singularity is called a *singularity of representation*. M. Renaud [5] has proposed a representation based on the parameters of Olinde Rodrigues (POR), also called Euler parameters. Pursuing this approach, we demonstrated [4] the existence of 4 systems of 3 POR for which some important mathematical operators are invariant, and such that the end effector rotation can be described by one of them for any configuration. By continuously selecting among the 4 systems, we obtain a representation using a non-fixed set of 3 independent parameters. The invariance property makes the selection changes in this representation transparent to the various associated mathematical models.

* The reported work was largely performed at the Centre d'Etude et de Recherche de Toulouse, France.

4. End Effector Dynamic Model

Let us consider the case of non-redundant manipulators, *i.e.* $n = m_0$, and use a set of independent parameters, *i.e.* operational coordinates, to represent the end effector configuration. Let $\underline{q}_i$ and $\bar{q}_i$ be respectively the minimal and maximal bounds of q_i. The movement of the point q in joint space is confined to the hyperparallelepiped:

$$D_q = \prod [\underline{q}_i, \bar{q}_i] \tag{6}$$

In the operational space R^{m_0}, the movement of the point x is within the domain D_x deduced from (1) and (6):

$$D_x = G(D_q) \tag{7}$$

Let $D_q^\dagger$ be the domain obtained from D_q by excluding the singular points in the kinematic model (2) and such that the vector function G of (1) is one-to-one. Let $D_x^\dagger$ designate the domain:

$$D_x^\dagger = G(D_q^\dagger) \tag{8}$$

In $D_x^\dagger$, the independent parameters $x_1, x_2, \ldots, x_{m_0}$ constitute a set of configuration parameters for the manipulator. Therefore, they constitute a set of generalized coordinates. The manipulator's kinetic energy is a quadratic form of the generalized velocities, since the articulated system is supposed holonomic. Its expression in terms of the operational coordinates and velocities is:

$$T_x(x, \dot{x}) = \frac{1}{2}\dot{x}^T \Lambda(x)\dot{x} \tag{9}$$

where $\Lambda(x)$ designates the symmetric matrix of the quadratic form. Lagrange's equations are:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = F_i \;;\quad (1 \le i \le m_0) \tag{10}$$

where the Lagrangian $L(x, \dot{x})$ is:

$$L(x, \dot{x}) = T_x(x, \dot{x}) - U(x) \tag{11}$$

$U(x)$ represents the gravity potential energy and F_i is the i^{th} operational force. Let $\eta(x, \dot{x})$ be the column matrix:

$$\eta(x, \dot{x}) = \left[\frac{\partial T_x}{\partial x_1} \cdots \frac{\partial T_x}{\partial x_{m_0}}\right]^T \tag{12}$$

Let $\mu(x, \dot{x})$ be the column matrix:

$$\mu(x, \dot{x}) = \dot{\Lambda}(x)\dot{x} - \eta(x, \dot{x}) \tag{13}$$

and

$$p(x) = \left[\frac{\partial U(x)}{\partial x_1} \cdots \frac{\partial U(x)}{\partial x_{m_0}}\right]^T \tag{14}$$

The end effector dynamic model of a non-redundant manipulator in the domain $D_x^\dagger$ may now be expressed using equations (13) and (14):

$$\Lambda(x)\ddot{x} + \mu(x, \dot{x}) - p(x) = F \tag{15}$$

$\Lambda(x), \mu(x, \dot{x})$ and $p(x)$ represent respectively the kinetic energy matrix, the centrifugal and Coriolis forces column matrix and the gravity forces column matrix. F is the m_0 operational forces column matrix.

5. Decoupling of End Effector Motions

The dynamic model (15) provides a description of the dynamic behavior of the end effector motions in operational space. The control of the manipulator for a desired motion in this space becomes feasible by selecting F as control vector. In order to produce this control vector of operational forces, specific forces Γ must be applied with joint-based actuators. The relationship between F and the joint forces Γ may be obtained by exploiting the identity between the virtual work of F in an elementary displacement δx and the virtual work of Γ in the corresponding displacement δq, according to the *virtual work principle*. Using equation (2) this leads to:

$$\Gamma = J^T(q)\, F \tag{16}$$

The decoupling of the end effector motion in the domain $D_x^\dagger$ of the operational space is achieved by employing the following structure of control:

$$F = \Lambda(x)F_m + \mu(x, \dot{x}) - p(x) \tag{17}$$

where F_m represents the input to the decoupled system. It is computed as a function of the desired motion (denoted by the subscript $_d$) and the actual motion, as shown in Figure 1 where K and ξ represent the $m_0 \times m_0$ constant gain matrices.

6. Derivation of the Dynamic Model Coefficient Matrices

Exploiting the identity between the kinetic energy quadratic forms $T_x(x, \dot{x})$ and $T_q(q, \dot{q})$ defined by:

$$T_q(q, \dot{q}) = \frac{1}{2}\dot{q}^T A(q)\dot{q}; \tag{18}$$

as well as relation (2), straightforward manipulation yields:

$$A(q) = J^T(q)\Lambda(x)J(q) \tag{19}$$

In the domain $D_q^\dagger$, the matrix $\Lambda(x)$ is given by:

$$\Lambda(x) = J^{-T}(q)A(q)J^{-1}(q) \tag{20}$$

Expansion of the quantities appearing in equation (13) and use of the above expression for Λ lead to:

$$\begin{aligned} \dot{\Lambda}(x)\dot{x} &= J^{-T}(q)\dot{A}(q)\dot{q} - \Lambda(q)h(q, \dot{q}) + \dot{J}^{-T}(q)A(q)\dot{q} \\ \eta(x, \dot{x}) &= J^{-T}(q)l(q, \dot{q}) + \dot{J}^{-T}(q)A(q)\dot{q} \end{aligned} \tag{21}$$

where

$$h(q, \dot{q}) = \dot{J}(q)\dot{q}\,; \tag{22}$$

and the element l_i of the column matrix $l(q, \dot{q})$ is defined as:

$$l_i = \frac{1}{2}\dot{q}^T A_{q_i}(q)\dot{q} \;;\; (1 \le i \le n) \tag{23}$$

the subscript $_{q_i}$ indicates the partial derivative with respect to the i^{th} joint coordinate. Further observing that:

$$b(q, \dot{q}) = \dot{A}(q)\dot{q} - l(q, \dot{q}) \tag{24}$$

the column matrix $\mu(x, \dot{x})$ may be written as:

$$\mu(x, \dot{x}) = J^{-T}(q)b(q, \dot{q}) - \Lambda(q)h(q, \dot{q}) \tag{25}$$

The matrix $p(x)$ of gravity forces is given by:

$$p(x) = J^{-T}(q)g(q) \tag{26}$$

The evaluation of the matrices Λ, μ, p in the foregoing expressions having been obtained in terms of the joint coordinates, the domain $D_x^\dagger$ of applicability of the dynamic model (15) may be extended to the domain D_x^* defined by:

$$D_x^* = G(D_q^*) \tag{27}$$

where D_q^* is the domain resulting from D_q of (6) when the singular points in the kinematic model are excluded. Indeed, the restriction to a domain where G is one-to-one then becomes unnecessary.

7. New Control Law Structure

Using equations (16), (25), and (26) the vector Γ corresponding to the control law (17) becomes:

$$\Gamma = J^T(q)\Lambda(q)F_m(q) + \tilde{b}(q, \dot{q}) - g(q) \tag{28}$$

with

$$\tilde{b}(q, \dot{q}) = J^T(q)\mu(q, \dot{q}); \tag{29}$$

which may be written in the form:

$$\tilde{b}(q, \dot{q}) = \tilde{B}(q)[\dot{q}\dot{q}] + \tilde{C}(q)[\dot{q}^2] \tag{30}$$

The $n \times n(n-1)/2$ matrix $\tilde{B}(q)$ and $n \times n$ matrix $\tilde{C}(q)$ are given by:

$$\begin{aligned} \tilde{B}(q) &= B(q) - J^T(q)\Lambda(q)H_1(q) \\ \tilde{C}(q) &= C(q) - J^T(q)\Lambda(q)H_2(q) \end{aligned} \tag{31}$$

where the matrices $H_1(q)$ and $H_2(q)$ have respectively the dimensions $n \times n(n-1)/2$ and $n \times n$ and are defined by:

$$h(q, \dot{q}) = H_1(q)[\dot{q}\dot{q}] + H_2(q)[\dot{q}^2] \tag{32}$$

The control law (28) becomes:

$$\Gamma = J^T(q)\Lambda(q)F_m + \tilde{B}(q)[\dot{q}\dot{q}] + \tilde{C}(q)[\dot{q}^2] - g(q) \tag{33}$$

8. Singular Configurations

The end effector dynamic parameters (EEDP's) do not all tend to infinity when the system's configuration tends to a singular configuration *i.e.*, when $det[J(q)] = 0$. Indeed, a *singular* *configuration* is a configuration q from which the end effector cannot move along or rotate around any given direction of the Cartesian space. In such a configuration, the manipulator's mobility locally decreases (the degree of mobility of the end effector is the rank of the basic Jacobian [6]. To a singular configuration corresponds a singular direction attached to the end effector. It is for that direction in fact that the effector presents an infinite inertial mass for a displacement or an infinite inertia for a rotation. Its movements will thus remain free in the sub-space orthogonal to this direction. The boundary values of the EEDP's in such a configuration are then obtained by writing their expressions in a coordinate system having the singular direction as one of its axes. In a singular configuration the transposed matrix $J^T(q)$ used in force mapping (16) exists; the inverse Jacobian required in the arm solution does not. Let us examine the system's asymptotic stability in a singular configuration. According to the control law (33), the manipulator is subject to the dissipative forces Γ_{dis} due to an $\dot{x}$ term in F_m:

$$\Gamma_{dis} = D(q)\dot{q} \tag{34}$$

$$D(q) = \xi_0 J^T(q)\Lambda(q)J(q) \tag{35}$$

where ξ_0 is a positive constant ($\xi = \xi_0 I$). Except in the singular configurations, the rank of the $n \times n$ matrix $D(q)$ is n and it is positive definite. Its rank decreases in the singular positions, and $D(q)$ simply becomes *non-negative definite*. Although the stability condition of the articulated system [7]:

$$\dot{q}^T D(q)\dot{q} \leq 0 \tag{36}$$

is still met, the system could describe, in this case, movements that are a solution of the equation [8]:

$$\dot{q}^T D(q)\dot{q} = 0 \tag{37}$$

Having an n-dimensional control vector of joint forces, the system's asymptotic stabilization may be achieved by the addition of dissipative forces proportional to $\dot{q}$.

9. Implementation Problems

Obtaining the EEDP's, elements of the $\Lambda(q)$, $\tilde{B}(q)$ and $\tilde{C}(q)$ matrices, in an explicit form requires complex analytical manipulations that could be performed easily by an interactive program for the automatic generation of these expressions. Research in this direction is currently planned. Storage and real-time look-up of the EEDP's in the configuration space appears as an interesting alternative to the analytical form calculation. However, for the EEDP's storage, a much larger space is required in comparison with the already large joint dynamic parameters storage space [9]. Indeed, in the end effector case, $\Lambda(q)$, $\tilde{B}(q)$ and $\tilde{C}(q)$ are a function of $n-1$ joint coordinates whereas in the joint case, $A(q)$, $B(q)$ and $C(q)$ are generally a function of $n-2$ coordinates [4]. Thus, for the conditions considered by Raibert (*i.e*, 8 quantification levels) 2000KB are required instead of 250KB.

10. Dynamic Properties

A detailed analysis of the EEDP expressions for various operational coordinate systems has been carried out and several basic properties were found [4]. They enable the control law (33) to be expressed in a form such that its parameters are a function of only $n-2$ joint coordinates. For example, let us consider the end effector dynamic model associated with the 3 POR representation. Let q^* be the vector $(q_2\ q_3 \ldots q_{n-1})^T$ and $J_0(q^*)$ be the basic Jacobian matrix written in the coordinate system $\mathcal{R}_1$ attached to the first link. It can be shown that the Jacobian associated with the POR may be obtained from $J_0(q^*)$ by:

$$J(q) = \Omega(q)J_0(q^*) \tag{38}$$

where $\Omega(q)$ is an $m_0 \times m_0$ matrix given in terms of the rotation matrix between the fixed referential frame $\mathcal{R}_0$ and $\mathcal{R}_1$, and of an operator associated with the 3 POR [4]. The control law (33) then becomes:

$$\Gamma = J_0^T(q^*)\Lambda_0(q^*)F^*_m + \tilde{B}_0(q^*)[\dot{q}\dot{q}] + \tilde{C}_0(q^*)[\dot{q}^2] - \Pi(q^*)\nu(q_1) \tag{39}$$

with

$$F^*_m = \Omega^{*-1}(q)[F_m + \frac{\omega^2}{4}\chi]. \tag{40}$$

The matrices $\Lambda_0(q^*)$, $\tilde{B}_0(q^*)$ and $\tilde{C}_0(q^*)$ have the same definition as in (20) and (31,32) with $J^T(q)$ in (20,22,31) being replaced by $J_0^T(q^*)$. $\Pi(q^*)$ and $\nu(q_1)$ are $n \times 3$ and 3×1 matrices such that:

$$g(q) = \Pi(q^*)\nu(q_1) \tag{41}$$

ω is the angular velocity vector and $\chi = (0\,0\,0\,x_4\,x_5\,x_6)^T$. The expression $\frac{\omega^2}{4}\chi$ in (40) results from the relationship between the angular acceleration and the POR acceleration. The structure of the control system (41) is shown in figure 2. The EEDP's storage space is thus reduced to the same magnitude as the joint dynamic parameters storage space. However, rather than using a large amount of memory, a better compromise between *memory, computing volume and accuracy* may be obtained by multivariable approximation techniques.

11. Multivariable Approximation

Let $\Xi^i(q^*)$ designate the 4-variable function representing the i^{th} EEDP, and Ξ^i the sought approximation polynomial. Considering Ξ^i as a function of some coordinates and parameterized with respect to the others, leads to different solutions to the approximation. For each solution the Ξ^i polynomial coefficients are parametrized with respect to a subset of the coordinates and stored in the related subspace. A comparative study that considered different polynomial development bases for various variable selections has been carried out. In regards to the parametrization problem, this study showed that a good compromise would be to consider Ξ^i to be a function of q_2 and q_3, and parametrized with respect to q_4 and q_5, *i.e.* $\Xi^i_{q_4,q_5}(q_2,q_3)$. The proposed solution consists of finding the $(k+1)(k+2)/2$ coefficients α_{ij} of the approximation defined by:

$$\Xi^i_{q_4,q_5}(r_0, z_0) = \sum_{i=0}^{k}\sum_{j=i}^{k} \alpha_{ij} r_0^{(i)} z_0^{(j)} \tag{42}$$

This approximation will be called *to the order k*. r_0 represents the length of the vector linking the origin of the frame system $\mathcal{R}_1$ to the origin of $\mathcal{R}_4$ and z_0 represents its projection on the z axis of $\mathcal{R}_0$. The variables (r_0, z_0) rather than (q_2, q_3) were chosen because of their advantage regarding the approximation accuracy. However, (r_0, z_0) may have identical values in different manipulator configurations. An important property concerning the EEDP values in certain symmetrical arm configurations [4] resolves this non-bijectivity problem. The approximation order k may be limited to 3. In the case of the robot Renault $V80$, for example, this produces a global relative error (mean squares) of less than 1%. In comparison with the storage-look-up method (with 8 quantification levels), this solution enables a reduction of 84%in memory space and increases the approximation accuracy 5 times, with the cost of 9 multiplications and 9 additions per parameter. For a 6 *dof* manipulator, 38KB are necessary to store the polynomial coefficients approximating its (about 60) EEDP's. The computing volume needed to obtain the EEDP's from the approximation coefficients is 540 multiplications and 540 additions.

12. Redundant Representations

The EEDP's in the control law (39) are linked to the basic Jacobian matrix. It therefore becomes easy to deduce a control law for the rotation vector ω. When the end effector desired orientation variation is described by a redundant representation, it may be replaced by the corresponding variation of the angular rotation vector which is controllable. The problem to be solved is then to find the vectors ω, $\dot{\omega}$ and the angular error vector $\delta\phi$ corresponding to the desired rotation. This is an inversion problem. We show in [4] the existence of a *left inverse* whose calculation is straightforward for various redundant representations. This easily resolves the previous problem.

13. Redundant Manipulators

In the case of redundancy, the operational coordinates can't constitute a generalized coordinate system since their number is less than the manipulator *dof*, *i.e.*, $m_0 < n$. Therefore, the manipulator's dynamic behaviour cannot be described by a dynamic model worked out in the operational space. However, dynamic equations of motion in this space may be obtained for the end effector. Using the dynamic model (4-5) and the relation:

$$\ddot{x} = J(q)\ddot{q} + h(q, \dot{q}) \tag{43}$$

these equations are [4]:

$$\Lambda_r(x)\ddot{x} + \mu_r(x, \dot{x}) - p_r(x) = F \tag{44}$$

with

$$\begin{aligned} \Lambda_r(q) &= [J(q)A^{-1}(q)J^T(q)]^{-1} \\ \mu_r(q, \dot{q}) &= \bar{J}^T(q)b(q, \dot{q}) - \Lambda_r(q)h(q, \dot{q}) \\ p_r(q) &= \bar{J}^T(q)g(q) \end{aligned} \tag{45}$$

where

$$\bar{J}(q) = A^{-1}(q)J^T(q)\Lambda_r(q) \tag{46}$$

$\bar{J}(q)$ is actually a right pseudo-inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's kinetic energy. An asymptotic stability analysis similar to that made in section 7 shows that in the redundant case the control law must include supplementary dissipative forces proportional to $\dot{q}$.

14. Obstacles

The operational space control approach enabled the development of a unique obstacle avoidance scheme based on the use of potential functions around obstacles, rather than actually planning paths. The philosophy of this approach can be schematically described as follows: *The manipulator moves in a field of forces. The position to be reached is an attractive pole for the end effector, and the obstacles are repulsive surfaces for the manipulator parts*[10]. Obstacles are described by composition of *primitives*. Analytic equations representing envelopes best approximating the primitives' shapes have been developed (parallelepiped, cone, cylinder, *etc.*). The control of a given point of the manipulator vis-à-vis an obstacle is achieved by submitting it to a *Force Inducing an Artificial Repulsion from the Surface* (FIRAS, from the french). These forces are created by an artificial potential field V obtained as a function of the normal distance to the obstacle's approximating surface ρ[4]:

$$V(\rho) = \begin{cases} \left(\frac{1}{\rho^2} - \frac{1}{\rho_0^2}\right)^2, & \text{if } |\rho| < |\rho_0|; \\ 0, & \text{if } |\rho| > |\rho_0|; \end{cases} \tag{47}$$

where ρ_0 represents the limit distance of the potential field influence. ρ is easily obtained using a variational procedure. Considering the small amount of calculation needed, this method allows obstacle avoidance to occur in real time as an integral part of the servo-control.

15. Applications

An experimental manipulator programming system "COSMOS" has been designed at the Stanford A.I. Laboratory for implementation of the presented control method for the PUMA arms. In the absence of an effective force control, a simplified end effector dynamic model of the PUMA 600 arm is used. Demonstrations of compliance of the PUMA end effector in a line, plane, circle and sphere, and of motions with obstacles (including mobile obstacles) detected by an MIC vision module have been performed. COSMOS, written in PASCAL, is implemented on a PDP11/45 computer. The servo rate is 50HZ.

16. Conclusion

The end effector dynamic model constitutes an effective tool for the analysis and control of manipulator behavior in operational space. A new control law structure to decouple the end effector motion has been developed. Using several basic dynamic properties and multivariable approximation techniques, the real-time implementation problem has been solved. The amount of calculation needed to decouple the end effector motion has been shown to be of the same magnitude as that for joint motion decoupling, while allowing the arm solution to be replaced by a simple force transformation; this represents a large reduction in computation. In addition, the singularity problem is avoided. The formulation of the control law in terms of the basic Jacobian matrix permits the extension of this control method to redundant representations of the rotation. It has also been extended to redundant manipulators, and would be highly suitable to application in multi-chained mechanisms [11]. Collision avoidance, generally treated at the highest level of control, has been demonstrated here to be an effective component of low-level real-time control. By its nature, operational space control is well suited to both the stipulation and satisfaction of geometric constraints on arm movement, and the control of applied forces. This approach, and more generally all dynamic approachs, require effective force control. Incorporation of joint force sensing feedback is indispensable.

Acknowledgments

Thanks to Raja Chatila, Harlyn Baker, John Craig and Bernie Roth for their suggestions and aid in preparation of the manuscript.

REFERENCES

[1] O. Khatib, M. Llibre et R. Mampey," Fonction *Décision-Commande d'un robot manipulateur*". *Rapport Scientifique*, No. 2/7156 DERA-CERT, Toulouse, Juillet 1978.

[2] M. Renaud, and J. Zabala-Iturralde, "Robot Manipulator Control". *Proc. 9th ISIR*, Washington, PP. 463-475, March 1979

[3] J.Y.S. Luh, M.W. Walker, and R. Paul, "Resolved acceleration control of mechanical manipulators". *IEEE Trans. On Automatic Control*, No. 3, pp. 468-474, June 1980.

[4] O. Khatib, "Commande dynamique dans l'espace opérationnel des robots manipulateurs en présence d'obstacles". *Thèse de Docteur-Ingénieur*, No. 37 , ENSAE, Toulouse, Dec. 1980.

[5] M. Renaud, "Contribution à la modélisation et à la commande dynamique des robots manipulateurs". *Thèse d'Etat*, Spécialité Automatique, Toulouse, Sep. 1980.

[6] A. Fournier, "Génération de mouvements en robotique. Application des inverses généralisées et des pseudo inverses". *Thèse d'Etat*, Mention Science, Montpellier, Avril 1980.

[7] D.L. Mingori, "A stablity theorem for mechanical systems with constraint damping". *Journal of Applied Mechanics*, *Trans. of the ASME*, pp. 253-258,1970

[8] V.V. Rumiantsev, "On the optimal stabilisation of controlled systems, *PMM*, vol. 34, No. 3, pp. 440-456,1970.

[9] M.H. Raibert, and B.K. Horn, "Manipulator control using the configuration space method". *Industrial Robot*, vol. 5, pp. 69-73, June 1978.

[10] O. Khatib, and J.F. Le Maitre, "Dynamic control of manipulators operating in a complex environment". 3rd *CISM-IFToMM*, PP. 267-282, Udine, Sep. 1978.

[11] J.K. Salisbury, and J.J. Craig, "Articulated Hands: Kinematic and Force Control Issues", *International Journal of Robotics Research*, Vol. 1,No. 1, pp. 4-17, Spring 1982.

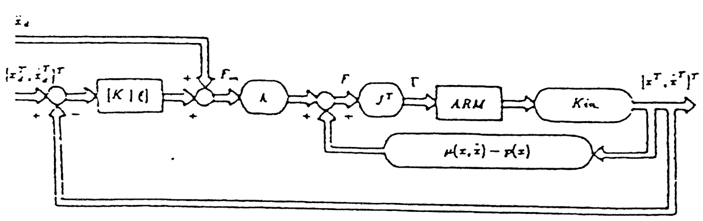

Figure 1. End effector motion decoupling.

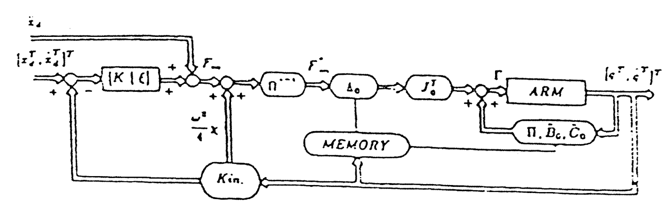

Figure 2. New control system implementation (POR).