The Operational Space Formulation in Robot Manipulator Control

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Abstract

The paper presents a radically new approach to real-time dynamic control and active force control of manipulators. In this approach the manipulator control problem is reformulated in terms of direct control of manipulator motion in operational space, the space in which the task is originally described, rather than controlling the task's corresponding joint space motion obtained after geometric and kinematic transformation. The control method is based on the construction of the manipulator end effector dynamic model in operational space. The generalized end effector forces are selected as the command vector in a control system designed for the decoupling of the end-effector motion in operational space. Forces are actually produced by the manipulator joint-based actuators. These joint forces are obtained by a simple force transformation from the operational space command vector. Dynamic control in operational space constitutes a unified approach to motion and active force control using a unified force command vector. It is also a powerful method for the control of redundant manipulators, and is the basis for the application to a robot arm of a unique real-time obstacle avoidance approach based on the artificial potential field concept. A two-level control architecture has been designed to improve the system real-time performance. This method has been implemented in the COSMOS system for a PUMA 560 robot using two processors. Compliance, contact, sliding and insertion operations using wrist and finger sensing have been demonstrated.

1. Introduction

Conventional manipulator control, which provides only linear feedback compensation to control joint positions independently, cannot meet the high accuracy and performance needed in precision manipulator tasks. Addressing this problem, much research has been directed at developing and modelling the dynamic equations of joint motion. Typical models relate joint variables to generalized torques and by necessity force the resulting control scheme to have two levels:

- The first level requires coordinate transformations to convert the task description from operational space to joint space;
- The second level makes use of the arm's dynamic model to calculate generalized force commands.

This first stage of transforming the task description is time consuming and prone to problems near kinematic singularities. Additionally, dealing with the dynamic compensation problem leads to high computational complexity in real-time control. In fact, the very approach of joint space control is ill-suited for active force control, an ability which is crucial in robot assembly tasks.

Robot collision avoidance, on the other hand, has typically been a component of higher levels of control in hierarchical robot control systems. It has been treated as a planning problem, and research in this area has focused on the development of collision-free path planning algorithms.

The *operational space formulation* presented in this paper enables the development of a unified approach to real-time dynamic control and active force control of manipulators, and a real-time collision avoidance method based on the artificial potential field concept. This formulation has its roots in the work on end-effector motion control and obstacle avoidance [Khatib and Le Maitre 78b] that we implemented for an MA23 manipulator in 1978. It has been formalized by constructing its basic tool, the end-effector equations of motion in operational space [Khatib 80].

2. Operational Space Formulation

Let $x_1, x_2, \ldots, x_m$ be the *m configuration parameters* of the end-effector, describing its position and orientation in a frame of reference R_0. An *operational coordinate system* is a set $\mathbf{x}$ of m_0 *independent* end-effector

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configuration parameters. The geometric and kinematic models of a manipulator are:

$$\mathbf{x} = \mathbf{G}(\mathbf{q}); \tag{1}$$

$$\dot{\mathbf{x}} = J(\mathbf{q})\ \dot{\mathbf{q}}; \tag{2}$$

where $\mathbf{q}$ is the vector of the n joint coordinates, and $J(\mathbf{q})$ the Jacobian matrix.

Let us first consider the case of non-redundant manipulators, *i.e.* $n = m_0$, and use a set of independent parameters, *i.e.* operational coordinates, to represent the end effector configuration. Let $\underline{q}_i$ and $\overline{q}_i$ be respectively the minimal and maximal bounds of q_i. The point $\mathbf{q}$ in joint space is confined to the hyperparallelepiped:

$$D_q = \prod_{i=1}^{n} [\underline{q}_i, \overline{q}_i]. \tag{3}$$

Let $\tilde{D}_q$ be the domain obtained from D_q by excluding the singular points in the kinematic model (2) and such that the vector function $\mathbf{G}$ of (1) is one-to-one. Let $\tilde{D}_x$ designate the domain:

$$\tilde{D}_x = \mathbf{G}(\tilde{D}_q). \tag{4}$$

For a non-redundant manipulator, the independent parameters $x_1, x_2, \ldots, x_{m_0}$ form a set of configuration parameters in the domain $\tilde{D}_x$ of the operational space and thus constitute a system of generalized coordinates. The kinetic energy of the holonomic articulated mechanism is a quadratic form of the generalized velocities:

$$T(\mathbf{x}, \dot{\mathbf{x}}) = \frac{1}{2}\dot{\mathbf{x}}^T \Lambda(\mathbf{x})\dot{\mathbf{x}}; \tag{5}$$

where $\Lambda(\mathbf{x})$ designates the $m_0 \times m_0$ symmetric matrix of the quadratic form, *i.e.* the kinetic energy matrix.

Using the Lagrangian formalism, the end-effector equations of motion are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{x}}}\right) - \frac{\partial L}{\partial \mathbf{x}} = \mathbf{F}; \tag{6}$$

where the Lagrangian $L(\mathbf{x}, \dot{\mathbf{x}})$ is:

$$L(\mathbf{x}, \dot{\mathbf{x}}) = T(\mathbf{x}, \dot{\mathbf{x}}) - U(\mathbf{x}); \tag{7}$$

and $U(\mathbf{x})$ represents the potential energy due to gravity. $\mathbf{F}$ is the operational force vector. Let $\mathbf{p}(\mathbf{x})$ be the vector of gravity forces:

$$\mathbf{p}(\mathbf{x}) = \frac{\partial U(\mathbf{x})}{\partial \mathbf{x}}. \tag{8}$$

The end-effector equations of motion can be developed [Khatib 80, Khatib 83] and written in the form:

$$\Lambda(\mathbf{x})\ddot{\mathbf{x}} + \mu(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}) = \mathbf{F}; \tag{9}$$

where $\mu(\mathbf{x}, \dot{\mathbf{x}})$ represents the vector of end-effector centrifugal and Coriolis forces given by:

$$\mu_i(\mathbf{x}, \dot{\mathbf{x}}) = \dot{\mathbf{x}}^T \Pi_i(\mathbf{x})\dot{\mathbf{x}}; \quad (i = 1, \ldots, m_0); \tag{10}$$

where the components of the $m_0 \times m_0$ matrices $\Pi_i(\mathbf{x})$ are the Christoffel symbols $\pi_{i,jk}$ given as a function of the partial derivatives of $\Lambda(\mathbf{x})$ *w.r.t.* the generalized coordinates $\mathbf{x}$ by:

$$\pi_{i,jk} = \frac{1}{2}\left(\frac{\partial \lambda_{ij}}{\partial x_k} + \frac{\partial \lambda_{ik}}{\partial x_j} - \frac{\partial \lambda_{jk}}{\partial x_i}\right). \tag{11}$$

These forces are related to the manipulator end-effector, and are distinct from similar forces that arise when viewing the manipulator motion in joint space. The manipulator equations of motion in joint space are given by:

$$A(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \mathbf{\Gamma}; \tag{12}$$

where $\mathbf{b}(\mathbf{q},\dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$, and $\mathbf{\Gamma}$, respectively, represent the Coriolis and centrifugal, gravity, and generalized forces in joint space; and $A(\mathbf{q})$ is the $n \times n$ joint space kinetic energy matrix.

Exploiting the identity between the kinetic energy quadratic forms with respect to joint and operational velocities, we established [Khatib 80, Khatib 83] the relationships between the components of the joint space dynamic model and those of the operational space dynamic model. These are:

$$\begin{aligned} \Lambda(\mathbf{x}) &= J^{-T}(\mathbf{q})A(\mathbf{q})J^{-1}(\mathbf{q}); \\ \mu(\mathbf{x},\dot{\mathbf{x}}) &= J^{-T}(\mathbf{q})\mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) - \Lambda(\mathbf{q})\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}); \\ \mathbf{p}(\mathbf{x}) &= J^{-T}(\mathbf{q})\mathbf{g}(\mathbf{q}); \end{aligned} \tag{13}$$

where:

$$\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) = \dot{J}(\mathbf{q})\dot{\mathbf{q}}. \tag{14}$$

In the foregoing relations, $\Lambda, \mu, \mathbf{p}$ have been expressed in terms of joint coordinates. The domain $\tilde{D}_x$ of (4) can then be extended to the domain $\bar{D}_x$ defined by:

$$\bar{D}_x = \mathbf{G}(\bar{D}_q); \tag{15}$$

where $\bar{D}_q$ is the domain resulting from D_q of (3) by excluding the kinematic singularities in (2). Indeed, the restriction to a domain where $\mathbf{G}$ is one-to-one then becomes unnecessary.

The control of manipulators in operational space is based on the selection of $\mathbf{F}$ as a command vector. In order to produce this command, specific forces $\mathbf{\Gamma}$ must be applied with joint-based actuators. The relationship between $\mathbf{F}$ and the generalized joint forces $\mathbf{\Gamma}$ can be obtained by exploiting the identity between the virtual work of $\mathbf{F}$ in an elementary displacement $\delta\mathbf{x}$ and the virtual work of $\mathbf{\Gamma}$ in the corresponding displacement $\delta\mathbf{q}$, according to the *virtual work principle*. Using equation (2) this leads to:

$$\mathbf{\Gamma} = J^T(\mathbf{q})\ \mathbf{F}. \tag{16}$$

3. End-Effector Control

While in motion, a manipulator end-effector is subject to the highly nonlinear forces mentioned earlier. These nonlinearities can be compensated for by dynamic decoupling in operational space using the end-effector equations of motion (9).

Active force control has been treated within the framework of a joint space control system [Craig and Raibert 79, Salisbury 80]. However, wrist or finger sensing, the desired end-effector contact forces, and the end-effector stiffness and dynamics involved in this problem are closely linked to the end-effector and its dynamic behavior in operational space.

With end-effector dynamic decoupling, active force control can be naturally integrated into the control system by simply incorporating it into the operational force command vector:

$$\mathbf{F} = \mathbf{F}_m + \mathbf{F}_a; \tag{17}$$

where $\mathbf{F}_m$, $\mathbf{F}_a$ are the operational command vectors of motion and active force control, respectively. $\mathbf{F}_m$ is given by:

$$\mathbf{F}_m = \Lambda(\mathbf{x})\mathbf{F}^*_m + \mu(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}); \tag{18}$$

where $\mathbf{F}^*_m$ represents the command vector of the decoupled end-effector, which becomes equivalent to a *single unit mass*. For a desired motion of the end-effector, this command vector is given by:

$$\mathbf{F}^*_m = \ddot{\mathbf{x}}_d - k_p(\mathbf{x} - \mathbf{x}_d) - \xi(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d). \tag{19}$$

where $\mathbf{x}_d$, $\dot{\mathbf{x}}_d$ and $\ddot{\mathbf{x}}_d$ are respectively the desired position, velocity and acceleration of the end-effector. k_p and ξ are the position and velocity gains.

For tasks that involve large end-effector motion toward a goal position without path specification, the following command vector [Khatib, Llibre, and Mampey 78a, Khatib 85a]:

$$\mathbf{F}^{*}_{m} = -\xi(\dot{\mathbf{x}} - \nu\dot{\mathbf{x}}_d); \tag{20}$$

where:

$$\dot{\mathbf{x}}_d = \frac{k_p}{\xi}(\mathbf{x}_d - \mathbf{x}); \qquad \nu = min(1, \frac{V_{max}}{\sqrt{\dot{\mathbf{x}}_d^T\dot{\mathbf{x}}_d}}); \tag{21}$$

allows a straight line motion of the end-effector at a given speed V_{max}. The velocity vector $\dot{\mathbf{x}}$ is in fact controlled to be pointed toward the goal position while its magnitude is limited to V_{max}. The end-effector will then travel at that speed, in a straight line, except during the acceleration and deceleration segments. This scheme is particularly useful in the collision avoidance approach presented in §6. It is shown in Figure 1 in addition to other forces described in §6.

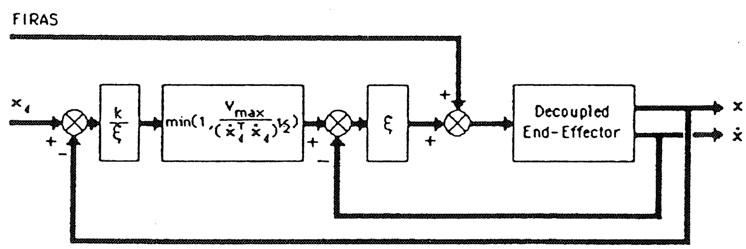

Figure 1. *End-effector Control for a Goal Position*

The operational command vector $\mathbf{F}$ in (17) is generated by the corresponding joint forces resulting from the force transformation (16). Let us designate by $[\dot{\mathbf{q}}\dot{\mathbf{q}}]$ and $[\dot{\mathbf{q}}^2]$ the $n(n-1)/2 \times 1$ and $n \times 1$ column matrices:

$$[\dot{\mathbf{q}}\dot{\mathbf{q}}] = [\dot{q}_1\dot{q}_2 \;\; \dot{q}_1\dot{q}_3 \cdots \dot{q}_{n-1}\dot{q}_n]^T; \qquad [\dot{\mathbf{q}}^2] = [\dot{q}_1^2 \;\; \dot{q}_2^2 \cdots \dot{q}_n^2]^T. \tag{22}$$

The joint forces corresponding to $\mu(\mathbf{x}, \dot{\mathbf{x}})$ can be written in the form:

$$\tilde{\mathbf{b}}(\mathbf{q}, \dot{\mathbf{q}}) = \tilde{B}(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \tilde{C}(\mathbf{q})[\dot{\mathbf{q}}^2]; \tag{23}$$

where $\tilde{B}(\mathbf{q})$ and $\tilde{C}(\mathbf{q})$ are, respectively, the $n \times n(n-1)/2$ and $n \times n$ matrices of the joint forces under the mapping into joint space of the end-effector Coriolis and centrifugal forces (see Appendix I). The joint torque vector corresponding to the operational space command vector (17) can then be developed as:

$$\Gamma = J^T(\mathbf{q})[\Lambda(\mathbf{q})\mathbf{F}^{*}_{m} + \mathbf{F}_a] + \tilde{B}(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \tilde{C}(\mathbf{q})[\dot{\mathbf{q}}^2] + \mathbf{g}(\mathbf{q}); \tag{24}$$

In order to simplify the notation, Λ has been also used here to designate the kinetic energy matrix when expressed as a function of the joint coordinate vector $\mathbf{q}$.

The dynamic decoupling of the end-effector can thus be obtained using the configuration-dependent dynamic coefficients $\Lambda(\mathbf{q})$, $\tilde{B}(\mathbf{q})$, $\tilde{C}(\mathbf{q})$ and $\mathbf{g}(\mathbf{q})$. By isolating these coefficients, end-effector dynamic decoupling and control can be achieved in a two-level control system architecture. The load of real-time computation of these coefficients can then be paced by the rate of configuration changes, which is much lower than that of the mechanism dynamics. Furthermore, the rate of computation of the end-effector position can be reduced by integrating an operational position estimator into the control system. Finally, the control system has the following architecture:

- A low rate ***parameter evaluation level***: updating the end-effector dynamic coefficents, the Jacobian matrix, and the geometric model.
- A high rate ***servo control level***: computing the command vector using the estimator and the updated dynamic coefficents.

By decoupling the end-effector motion, compliance in a given direction in the operational space is directly controlled by the position gain matrix. Active force control in a given direction is then simply achieved by setting the end-effector stiffness in that direction to zero and selecting the corresponding force servo using the matrix S (see Figure 2).

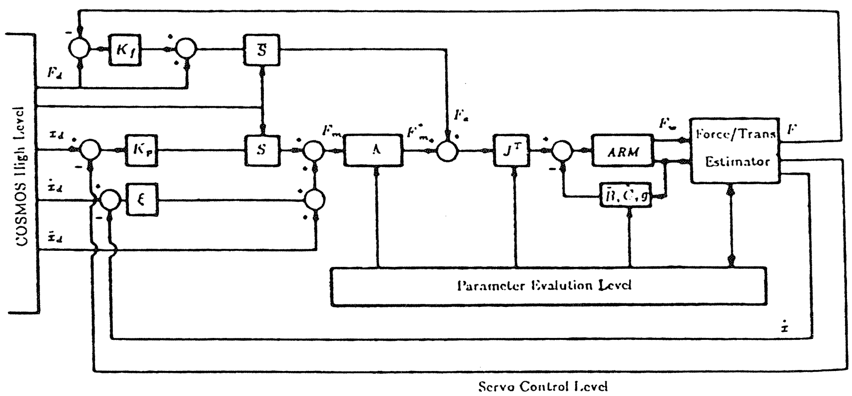

Figure 2. *Operational Space Control System Architecture*

4. Redundant Manipulators

For a redundant manipulator, the end-effector configuration parameters don't constitute a generalized coordinate system. Although the dynamic behavior of a redundant manipulator system cannot be represented by a dynamic model developed with respect to an end-effector configuration coordinate system, the dynamic behavior of the end-effector can still be described and its equations of motion in operational space can still be established. While these equations of motion can be used to achieve the control of the end-effector motions and active forces, the stabilization of the redundant mechanism must be based on the manipulator joint space dynamic model.

Using the dynamic model (12) and the relation:

$$\ddot{\mathbf{x}} = J(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}); \tag{25}$$

we established [Khatib 80] the following equations of motion for a redundant manipulator system:

$$\Lambda_r(\mathbf{x})\ddot{\mathbf{x}} + \mu_r(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{p}_r(\mathbf{x}) = \mathbf{F}; \tag{26}$$

where:

$$\begin{aligned} \Lambda_r(\mathbf{q}) &= [J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q})]^{-1}; \\ \mu_r(\mathbf{q}, \dot{\mathbf{q}}) &= \bar{J}^T(\mathbf{q})\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) - \Lambda_r(\mathbf{q})\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}); \\ \mathbf{p}_r(\mathbf{q}) &= \bar{J}^T(\mathbf{q})\mathbf{g}(\mathbf{q}); \end{aligned} \tag{27}$$

with:

$$\bar{J}(\mathbf{q}) = A^{-1}(\mathbf{q})J^T(\mathbf{q})\Lambda_r(\mathbf{q}). \tag{28}$$

$\bar{J}(\mathbf{q})$ is actually a pseudo-inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's kinetic energy.

The end-effector dynamic decoupling and control can be then obtained using a control system similar to that of (24):

$$\Gamma_r = J^T(\mathbf{q})[\Lambda_r(\mathbf{q})\mathbf{F}^*_m + \mathbf{F}_a] + \tilde{B}_r(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \tilde{C}_r(\mathbf{q})[\dot{\mathbf{q}}^2] + \mathbf{g}(\mathbf{q}); \tag{29}$$

where $\tilde{B}_r(\mathbf{q})$ and $\tilde{C}_r(\mathbf{q})$ have similar expressions to $\tilde{B}(\mathbf{q})$ and $\tilde{C}(\mathbf{q})$, with $\Lambda(\mathbf{q})$ being replaced by $\Lambda_r(\mathbf{q})$. Under this command vector, the manipulator is subject to the dissipative forces Γ_{dis} due to the $\dot{\mathbf{x}}$ term in $\mathbf{F}^*_m$:

$$\Gamma_{dis} = D(\mathbf{q})\dot{\mathbf{q}}; \tag{30}$$

with:

$$D(\mathbf{q}) = -\xi J^T(\mathbf{q})\Lambda_r(\mathbf{q})J(\mathbf{q}). \tag{31}$$

$D(\mathbf{q})$ is an $n \times n$ negative semi-definite matrix of rank m_0. Although the stability condition [Mingori 70]:

$$\dot{\mathbf{q}}^T D(\mathbf{q})\dot{\mathbf{q}} \leq 0; \tag{32}$$

of the articulated mechanical system (12) under the previous command is satisfied, this redundant mechanism can still describe movements that are solutions of the equation [Rumiantsev 70]:

$$\dot{\mathbf{q}}^T D(\mathbf{q})\dot{\mathbf{q}} = 0; \tag{33}$$

Asymptotic stabilization of the system can be achieved by the addition of dissipative forces proportional to $\dot{\mathbf{q}}$ [Khatib 80]. These forces can be selected from the null space of the Jacobian matrix $J(\mathbf{q})$, in order to eliminate their influence on the end-effector behavior. Using the dynamic model (12), this corresponds to applying the additional stabilizing joint forces:

$$\Gamma_s = -\xi_q A(\mathbf{q})[I - \bar{J}(\mathbf{q})J(\mathbf{q})]\dot{\mathbf{q}}. \tag{34}$$

The application of Γ_s will not affect the resulting end-effector operational forces. By grouping the term $\xi_q A(\mathbf{q})\bar{J}(\mathbf{q})J(\mathbf{q})]\dot{\mathbf{q}}$ of (34) with the dissippative forces in $\mathbf{F}^*_m$, the joint force command vector can be written in the form:

$$\Gamma = J^T(\mathbf{q})[\Lambda_r(\mathbf{q})\mathbf{F}^*_m + \mathbf{F}_a] - \xi_q A(\mathbf{q})\dot{\mathbf{q}} + \tilde{B}_r(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \tilde{C}_r(\mathbf{q})[\dot{\mathbf{q}}^2] + \mathbf{g}(\mathbf{q}); \tag{35}$$

with:

$$\mathbf{F}^*_m = \ddot{\mathbf{x}}_d - k_p(\mathbf{x} - \mathbf{x}_d) - \xi(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) + \xi_q\dot{\mathbf{x}}; \tag{36}$$

for a desired motion, and:

$$\mathbf{F}^*_m = -\xi(\dot{\mathbf{x}} - \nu\dot{\mathbf{x}}_d) + \xi_q\dot{\mathbf{x}}; \tag{37}$$

for a goal position, where ν is defined as in (21). The null space of the Jacobian matrix can also be used in the optimization of additional objectives. In redundant manipulator control, the null space has been used in order to achieve goals such as avoidance of joint limits [Liegois 1977, Fournier 1980], obstacle avoidance [Hanafusa, Yoshikawa, and Nakamura 1981], or minimizing the actuator joint forces [Hollerbach and Suh 1985]. With the command vector (35), the matrix $D(\mathbf{q})$ in the new expression for the dissipative joint forces Γ_{dis} becomes:

$$D(\mathbf{q}) = -[(\xi - \xi_q)J^T(\mathbf{q})\Lambda_r(\mathbf{q})J(\mathbf{q}) + \xi_q A(\mathbf{q})]. \tag{38}$$

$D(\mathbf{q})$ is now a negative definite matrix and the closed loop system is asymptoticaly stable.

5. Singular Configurations

A *singular configuration* is a configuration $\mathbf{q}$ at which the end-effector loses the ability to move along or rotate about a given direction of the Cartesian space. In such a configuration, the manipulator's mobility locally decreases. To each singular configuration corresponds a singular "direction" in operational space. It is for that direction, in fact, that the effector presents infinite inertial mass for displacements or infinite inertia for rotations. Its movements remain free in the subspace orthogonal to this direction. The basic concept in our approach to the problem of kinematic singularities can be formulated as follows: *at a singular configuration, the manipulator can be treated as a redundant mechanism with respect to the end-effector motion in the subspace of operational space orthogonal to its singular direction.*

The equations of motion of the end-effector in that sub-space are similar to those of (26,27). The end-effector dynamic decoupling and control as well as the stabilization of the mechanism can be achieved similarly to the case of redundant manipulators in the previous section. In addition, joint forces from the Jacobian null space can be used to select the desired configuration of the manipulator among the various configuartions that the arm can take for a given motion of the end-effector.

6. The Artificial Potential Field Approach

The operational space control approach enabled the development of a unique obstacle avoidance scheme based on the use of potential functions around obstacles, rather than actual path planning. The philosophy of the artificial potential field approach can be schematically described as follows: *The manipulator moves in a field of forces. The position to be reached is an attractive pole for the end-effector, and obstacles are repulsive surfaces for the manipulator parts.*

Obstacles are described by composition of *primitives*. The control of a given point of the manipulator vis-à-vis an obstacle is achieved by submitting it to a *Force Inducing an Artificial Repulsion from the Surface* (FIRAS, from the French). These forces are created by an artificial potential field obtained as a function of the normal distance to the obstacle's surface. Collision avoidance for *moving obstacles* is obtained using a continuously time-varying potential field. The manipulator obstacle avoidance problem has been formulated in terms of *collision avoidance of links* [Khatib 85a], rather than points. Link collision avoidance is achieved by continuously controlling the link's closest point to the obstacle. The potential field approach has also been used to satisfy the manipulator internal joint constraints.

With this approach, the problem can be treated in two stages:

- at high level control, generating a global strategy for the manipulator's path in terms of intermediate goals (rather than finding an accurate collision-free path);
- at the low level, producing the appropriate commands to attain each of these goals, taking into account the detailed geometry and motion of manipulator and obstacle, and making use of real-time obstacle sensing (low level vision and proximity sensors).

7. Applications

This approach has been implemented in an experimental manipulator programming system COSMOS (Control in Operational Space of a Manipulator-with-Obstacles System). Using a PUMA 560, demonstrations of real-time end-effector motion and active force control operations have been performed. These include contact, slide, insertion, and compliance operations [Khatib 85b], as well as real-time collision avoidance with links and moving obstacles. In the current multiprocessor implementation (PDP 11/45 and PDP 11/60), the rate of the servo control level is 225 Hz while the coefficient evaluation level runs at 100 Hz.

8. Conclusion

We have presented the operational space formulation and constructed its basic tool, the end-effector equations of motion. A unified approach to motion and active force control of manipulator systems has been developed, and a two-level control system architecture has been designed in order to achieve higher real-time performance. We have also presented the extension of this formulation to redundant manipulator mechanisms, and have discussed the principal concepts in dealing with kinematic singularity problems. Further, we have described the artificial potential field approach for real-time collision avoidance. Through the results of our preliminary implementation, we have shown the operational space formulation to be an effective means of achieving high dynamic performance in both real-time motion control and active force control of robot manipulators, leading to an increased capability for more advanced assembly tasks in more complex environments.

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Appendix I

Let $B(\mathbf{q})$ and $C(\mathbf{q})$ be, respectively, the $n \times n(n-1)/2$, and $n \times n$, matrices of the joint space Coriolis and centrifugal forces defined by:

$$b(\mathbf{q}, \dot{\mathbf{q}}) = B(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + C(\mathbf{q})[\dot{\mathbf{q}}^2]; \quad (A1)$$

The matrices $\tilde{B}(\mathbf{q})$ and $\tilde{C}(\mathbf{q})$ of the joint forces under the mapping into joint space of the end-effector Coriolis and centrifugal forces $\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}})$ are related to $B(\mathbf{q})$ and $C(\mathbf{q})$ by:

$$\begin{aligned} \tilde{B}(\mathbf{q}) &= B(\mathbf{q}) - J^T(\mathbf{q})\Lambda(\mathbf{q})H_1(\mathbf{q}); \\ \tilde{C}(\mathbf{q}) &= C(\mathbf{q}) - J^T(\mathbf{q})\Lambda(\mathbf{q})H_2(\mathbf{q}); \end{aligned} \quad (A2)$$

where the matrices $H_1(\mathbf{q})$ and $H_2(\mathbf{q})$, respectively $n \times n(n-1)/2$ and $n \times n$, are:

$$\dot{J}(\mathbf{q})\dot{\mathbf{q}} = H_1(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + H_2(\mathbf{q})[\dot{\mathbf{q}}^2]. \quad (A3)$$