Constrained Motion and Redundancy in Robot Manipulator Control

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Abstract

In this paper, we discuss issues related to the description of manipulator end-effector tasks that involve constrained motion and active force control. A generalized task specification matrix is used in the development of a unified approach for motion and force control of manipulators in the operational space framework. We also present the extension of this formulation to redundant manipulator systems. The end-effector equations of motion in operational space of a redundant manipulator are established, and its behavior with respect to generalized joint forces is described. The end-effector is controlled by an operational space control system based on these equations of motion. Asymptotic stabilization of the mechanism is achieved by the use of dissipative joint forces selected from the null space of the Jacobian transpose matrix, consistent with the manipulator dynamics.

Introduction

Joint space dynamic models have been the basis for various approaches to dynamic control of manipulators. However, task specification for motion and applied forces, dynamics, and force sensing feedback are closely linked to the manipulator end-effector. The dynamic behavior of the end-effector is one of the most significant characteristics in evaluating the performance of robot manipulator systems. The issue of end-effector motion control has been investigated and algorithms resolving end-effector accelerations have been proposed [Takase 1977; Khatib, Llibre, and Mampey 1978; Hewit and Padovan 1978; Renaud, and Zabala-Iturralde 1979; Luh, Walker, and Paul 1980]. In manipulator force control, accommodation [Whitney 1977], joint compliance [Paul and Shimano 1976], active compliance [Salisbury 1980], passive compliance, and hybrid position/force control [Craig and Raibert 1979] are among the various methods that have been proposed.

Active force control has been generally based on kinematic considerations, and has been treated within the framework of joint space control systems. The *operational space formulation* [Khatib 1980, Khatib 1983], which establishes the end-effector equations of motion, has provided a framework in which this type of control [Khatib 1985, Khatib and Burdick 1986] can be naturally addressed.

Treated within the framework of joint space control systems, redundancy of manipulator mechanisms has been generally viewed as a problem of resolving the end-effector desired motion into joint motions in the sense of some criteria. The manipulator redundancy has been aimed at achieving goals such as the minimization of a quadratic criterion [Whitney 1969, Renaud 1975], the avoidance of joint limits [Liegois 1977, Fournier 1980], the avoidance of obstacles, [Hanafusa, Yoshikawa, and Nakamura 1981, Kircanski and Vukobratovic 1984, Espiau and Boulic 1985], kinematic singularities [Luh and Gu 1985], or the minimization of actuator joint forces [Hollerbach and Suh 1985].

By establishing the end-effector equations of motion and describing its behavior with respect to generalized joint forces, the control of a redundant manipulator is formulated here in terms of finding the operational end-effector forces and generalized joint forces that allow the end-effector to respond to the desired task, while ensuring asymptotic stabilization of the mechansim. The unified approach for motion and active force control is extended to redundant manipulator systems.

Generalized Task Specification Matrix

The end-effector motion and contact forces are among the most important components in the planning, description, and control of assembly operations of robot manipulators. The end-effector configuration is represented by a set of parameters specifying its position and orientation. In free motion operations, the number of *end-effector degrees of freedom*, m_0, is defined [Khatib 1980] as the number of independent parameters required to completely specify, in a frame of reference $\mathcal{R}_0$, its position and orientation. A set of such independent configuration parameters form a system of *operational coordinates*.

In constrained motion operations, the displacement and rotations of the end-effector are subjected to a set of geometric constraints. These constraints restrict the freedom of motion (displacements, and rotations) of the end-effector. It is clear that geometric constraints will affect only the freedom of motion of the end-effector, since static forces and moments at these constraints can still be applied. The number of degrees of freedom of the constrained end-effector is given by the difference between m_0 and the number of the independent equations specifying the geometric constraints, assumed to be holonomic.

An interesting description of the characteristics of end-effectors and their constraints uses a mechanical linkage representation [Fournier 1980, Mason 1981]. The end-effector, tool, or manipulated object forms, with the fixture or constrained object, a pair of two rigid bodies linked through a joint. A constrained motion task can be described, for instance, by a spherical, planar, cylindrical, prismatic, or revolute joint.

However, when viewed from the perespective of end-effector control, two elements of information are required for a complete description of the task. These are the vectors of total force and moment that are to be applied in order to maintain the imposed constraints, and the specification of the end-effector motion degrees of freedom and their directions.

Let $\mathbf{f}_d$ be the vector, in the frame of reference $\mathcal{R}_0(O, \mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$, of forces that are to be applied by the end-effector. The position freedom, if any, of the constrained end-effector will therefore lie in the subspace orthogonal to $\mathbf{f}_d$.

A convenient coordinate frame for the description of tasks involving constrained motion operations is a coordinate frame $\mathcal{R}_f(O, \mathbf{x}_f, \mathbf{y}_f, \mathbf{z}_f)$ obtained from $\mathcal{R}_0$ by a rotation transformation described by S_f such that $\mathbf{z}_f$ is in alignment with f_d. For tasks where the freedom of position motion is restricted to a single direction orthogonal to $\mathbf{f}_d$, one of the axes Ox or Oy will be selected in alignment with that direction, as shown for the task represented in Figure 1.

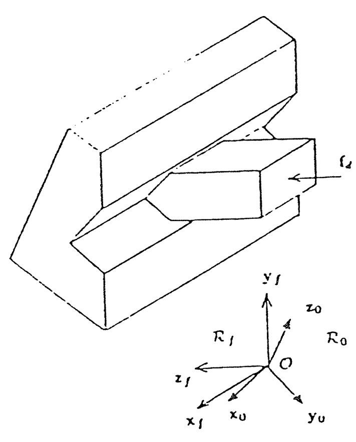

Figure 1: One-degree-of-freedom Position Motion.

To a task specified in terms of end-effector position motion and applied force in the coordinate frame $\mathcal{R}_f$, we associate the position specification matrix

$$\Sigma_f = \begin{pmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad (1)$$

where σ_x and σ_y are binary numbers assigned to the value 1 when a free motion is permitted following the Ox axis and/or the Oy axis respectively, and zero otherwise. The subspace of force control is described by the matrix associated with Σ_f and defined by

$$\overline{\Sigma}_f = I - \Sigma_f; \qquad (2)$$

where I designates the 3×3 identity matrix.

Let us now consider the case of tasks that involve constrained rotations and applied moments of the end-effector. Let τ_d be the vector, in the frame of reference $\mathcal{R}_0(O, \mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$, of moments that are to be applied by the end-effector, and $\mathcal{R}_r(O, \mathbf{x}_r, \mathbf{y}_r, \mathbf{z}_r)$ be a coordinate frame obtained from $\mathcal{R}_0(O, \mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$ by a rotation S_r that brings $\mathbf{z}_r$ into alignment with the task torque vector τ_d. In $\mathcal{R}_r$, the rotation freedom of the end-effector lies in the subspace spanned by $\{\mathbf{x}_r, \mathbf{y}_r\}$. Similarly to Σ_f and $\overline{\Sigma}_f$, we define with respect to $\mathcal{R}_r$ the rotation and moment specification matrices Σ_r and $\overline{\Sigma}_r$.

For tasks that involve end-effector motion (position and orientation) and applied forces (forces and torques) described in the frame of reference $\mathcal{R}_0$, we define *the generalized task specification matrix*

$$\Omega = \begin{pmatrix} S_f^T \Sigma_f S_f & 0 \\ 0 & S_r^T \Sigma_r S_r \end{pmatrix}; \qquad (3)$$

with which is associated the matrix

$$\tilde{\Omega} = \begin{pmatrix} S_f^T \overline{\Sigma}_f S_f & 0 \\ 0 & S_r^T \overline{\Sigma}_r S_r \end{pmatrix}. \qquad (4)$$

The construction of this matrix has been motivated by the aim of formulating efficiently the manipulator dynamic control architecture in a coordinate frame that is independent of the task specification. Control systems using specifications based on the matrices Σ_f and Σ_r will involve costly transformations of geometric, kinematic, and dynamic quantities to task coordinate frames.

Following the task specifications, Ω can be a constant, configuration-varying, or time-varying matrix. A non-constant generalized task specification matrix corresponds to specifications that involve changes in the direction of the applied force vector and/or moment vector, *e.g.* moving the end-effector while maintaining a normal force to a non-planar surface. Ω has been here expressed with respect to the frame of reference $\mathcal{R}_0$. For control systems implemented for tasks specified with respect to the end-effector coordinate frame, Ω will be specified with respect to that coordinate frame as well.

Unified Motion and Force Control

For a non-redundant manipulator, the end-effector equations of motion with respect to a system $\mathbf{x}$ of operational coordinates can be written as [Khatib 1980, Khatib 1983]

$$\Lambda(\mathbf{x})\ddot{\mathbf{x}} + \mu(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}) = \mathbf{F}; \tag{5}$$

where $\Lambda(\mathbf{x})$ designates the kinetic energy matrix, and $\mu(\mathbf{x},\dot{\mathbf{x}})$ represents the vector of end-effector centrifugal and Coriolis forces. $\mathbf{p}(\mathbf{x})$ and $\mathbf{F}$ are respectively the gravity and the generalized operational force vectors.

With respect to a system of n joint coordinates $\mathbf{q}$, the manipulator equations of motion in joint space can be written in the form

$$A(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \Gamma; \tag{6}$$

where $\mathbf{b}(\mathbf{q},\dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$, and Γ, represent the Coriolis and centrifugal, gravity, and generalized forces in joint space; and $A(\mathbf{q})$ is the $n \times n$ joint space kinetic energy matrix, which is related to $\Lambda(\mathbf{x})$ by

$$A(\mathbf{q}) = J^T(\mathbf{q})\Lambda(\mathbf{x})J(\mathbf{q}); \tag{7}$$

where $J(\mathbf{q})$ is the Jacobian matrix.

Free Motion Operations

The control of manipulators in operational space is based on the selection of $\mathbf{F}$ as a command vector. These generalized operational forces are generated by joint-based actuators. The generalized joint force vector Γ corresponding to F is given by

$$\Gamma = J^T(\mathbf{q})\ \mathbf{F}. \tag{8}$$

In free motion operations, the dynamic decoupling and motion control of the manipulator in operational space is achieved by selecting the control structure

$$\mathbf{F} = \Lambda(\mathbf{x})\mathbf{F}^*_m + \mu(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}); \tag{9}$$

where $\mathbf{F}^*_m$ represents the command vector of the decoupled end-effector.

Using equation (8), the joint forces corresponding to the operational command vector $\mathbf{F}$ in (9) can be written as

$$\Gamma = J^T(\mathbf{q})\Lambda(\mathbf{x})\mathbf{F}^*_m + \tilde{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}). \tag{10}$$

where $\tilde{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}})$ is the vector of joint forces under the mapping into joint space of the end-effector Coriolis and centrifugal force vector $\mu(\mathbf{x},\dot{\mathbf{x}})$.

Constrained Motion Operations

The matrix Ω defined above specifies, with respect to the frame of reference $\mathcal{R}_0$, the directions of motion (displacement and rotations) of the end-effector. Forces and moments are to be applied in or about directions that are orthogonal to those motion directions. These are specified by the matrix $\tilde{\Omega}$.

An important question related to the specifications of axes of rotations and applied moments is concerned with the compatibility between these specifications and the representation used in the description of the end-effector orientation. The specification of axes of rotation in the matrix Σ_r are only compatible with instantaneous angular rotations, which cannot be obtained from a set of orientation configuration parameters. Representations of the end-effector orientation such as Euler angles, direction cosines, or Euler parameters, are indeed incompatible with specifications provided by Σ_r.

However, instantaneous angular rotations have been used in the control of end-effector orientation. An angular rotation error vector $\delta\phi$ that corresponds to the error between the actual orientation of the end-effector and its desired orientation can be formed from the orientation description given by the selected representation [Luh, Walker, and Paul 1980, Khatib 1980]. With linear and angular velocities is associated the matrix $J_0(\mathbf{q})$, termed *the basic Jacobian*, defined independently of the particular set of parameters used to describe the end-effector configuration

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = J_0(\mathbf{q})\dot{\mathbf{q}}. \tag{11}$$

For end-effector motions specified in terms of Cartesian coordinates and instantaneous angular rotations, the dynamic decoupling and motion control of the end-effector can be achieved [Khatib 1980] by

$$\Gamma = J_0^T(\mathbf{q})\Lambda_0(\mathbf{x})\mathbf{F}^*_m + \tilde{\mathbf{b}}_0(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}); \tag{12}$$

where $\Lambda_0(\mathbf{q})$ and $\tilde{\mathbf{b}}_0(\mathbf{q},\dot{\mathbf{q}})$ are defined similarly to $\Lambda(\mathbf{q})$ and $\tilde{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}})$ with $J(\mathbf{q})$ being replaced by $J_0(\mathbf{q})$.

Similar control structures can be used to achieve dynamic decoupling and motion control with respect to descriptions using other representations for the orientation of the end-effector. This results from the possibility of expressing the Jacobian matrix $J(\mathbf{q})$ associated with a given representation of the end-effector orientation x_r as a function of the basic Jacobian by a relationship of the form

$$J(\mathbf{q}) = E_{x_r}J_0(\mathbf{q}); \tag{13}$$

where the matrix E_{x_r} is simply given as a function of x_r [Khatib 1980].

The unified operational command vector for end-effector dynamic decoupling, motion, and active force control can be written as

$$\mathbf{F} = \mathbf{F}_m + \mathbf{F}_a + \mathbf{F}_{ccg}; \tag{14}$$

where $\mathbf{F}_m$, $\mathbf{F}_a$, and $\mathbf{F}_{ccg}$ are the operational command vectors of motion, active force control, and centrifugal, Coriolis, and gravity forces given by

$$\begin{aligned} \mathbf{F}_m &= \Lambda_0(\mathbf{q})\Omega\mathbf{F}^*_m; \\ \mathbf{F}_a &= \tilde{\Omega}\mathbf{F}^*_a + \Lambda_0(\mathbf{q})\tilde{\Omega}\mathbf{F}^*_s; \\ \mathbf{F}_{ccg} &= \tilde{\mathbf{b}}_0(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}); \end{aligned} \tag{15}$$

where $\mathbf{F}^*_s$ represents the vector of end-effector velocity damping that acts in the subspace of force control. The joint force vector corresponding to $\mathbf{F}$ in (14), is

$$\Gamma = J_0^T(\mathbf{q})[\Lambda_0(\mathbf{q})(\Omega\mathbf{F}^*_m + \tilde{\Omega}\mathbf{F}^*_s) + \tilde{\Omega}\mathbf{F}^*_a] + \widetilde{\mathbf{b}_0}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}). \tag{16}$$

A more detailed description of the components involved in this control system, real-time implementation issues, and experimental results can be found in [Khatib 1985, Khatib and Burdick 1986].

Redundant Manipulators

The configuration of a redundant manipulator cannot be specified by a set of parameters that only describes the end-effector position and orientation. An independent set of end-effector configuration parameters, therefore, does not constitute a generalized coordinate system for a redundant manipulator, and the dynamic behavior of the entire redundant system cannot be represented by a dynamic model in coordinates only of the end-effector configuration. The dynamic behavior of the *end-effector* itself, nevertheless, can still be described, and its equations of motion in operational space can still be established.

Let us first consider the end-effector dynamic response to the application, on the end-effector, of an operational force vector $\mathbf{F}$. The joint forces corresponding to $\mathbf{F}$ are still given by (8). Using the dynamic model (6) and the relation

$$\ddot{\mathbf{x}} = J(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}); \tag{17}$$

we established [Khatib 1980] the following equations

$$\Lambda_r(\mathbf{q})\ddot{\mathbf{x}} + \mu_r(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{p}_r(\mathbf{q}) = \mathbf{F}; \tag{18}$$

where

$$\begin{aligned} \Lambda_r(\mathbf{q}) &= [J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q})]^{-1}; \\ \mu_r(\mathbf{q}, \dot{\mathbf{q}}) &= \bar{J}^T(\mathbf{q})\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) - \Lambda_r(\mathbf{q})\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}); \\ \mathbf{p}_r(\mathbf{q}) &= \bar{J}^T(\mathbf{q})\mathbf{g}(\mathbf{q}); \end{aligned} \tag{19}$$

with

$$\bar{J}(\mathbf{q}) = A^{-1}(\mathbf{q})J^T(\mathbf{q})\Lambda_r(\mathbf{q}). \tag{20}$$

$\bar{J}(\mathbf{q})$ is actually a generalized inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's instantaneous kinetic energy.

Equation (18) describes the dynamic behavior of the end-effector when the manipulator is submitted to a generalized joint force vector of the form (8). The $m \times m$ matrix $\Lambda_r(\mathbf{q})$ can be interpreted as a *pseudo-kinetic energy matrix* corresponding to the end-effector motion in operational space. $\mu_r(\mathbf{q}, \dot{\mathbf{q}})$ represents the Centrifugal and Coriolis forces acting on the end-effector, and $\mathbf{p}_r(\mathbf{q})$ the gravity force vector.

Let us now consider the case where an arbitrary joint force vector is applied to the redundant mechanism. Equation (18) can be rewritten as

$$\bar{J}^T(\mathbf{q})[A(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q})] = \mathbf{F}. \tag{21}$$

Substituting equation (6) yields

$$\mathbf{F} = \bar{J}^T(\mathbf{q})\Gamma. \tag{22}$$

The matrix $\bar{J}^T(\mathbf{q})$ describes how the joint space manipulator dynamic forces are reflected at the level of the end-effector.

Lemma

The unconstrained end-effector (18) is subjected to the operational force $\mathbf{F}$ *if and only if the manipulator (6) is submitted to the generalized joint force vector*

$$\Gamma = J^T(\mathbf{q})\mathbf{F} + [I_n - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\Gamma_o; \tag{23}$$

where I_n *the* $n \times n$ *identity matrix,* $\bar{J}(\mathbf{q})$ *is the matrix given in (20), and* Γ_o *is an arbitrary joint force vector.*

When the applied joint forces Γ are of the form (23), it is straightforward from equation (22) to verify that the only forces acting on the end-effector are the operational forces $\mathbf{F}$ produced by the first term in the expression of Γ. Joint forces of the form $[I_n - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\Gamma_o$ correspond in fact to a null operational force vector.

The uniqueness of (23) is essentially linked to the use of a generalized inverse $\bar{J}(\mathbf{q})$ that is consistent with the dynamic equations of the manipulator and end-effector. The form of the decomposition (23) itself is general. A joint force vector Γ can always be decomposed in the form (23), and various expressions for Γ associated with various generalized inverses $J(\mathbf{q})$ can be established.

Let $P(\mathbf{q})$ be a generalized inverse of $J(\mathbf{q})$ and let us submit the manipulator to the joint force vector

$$\Gamma = J^T(\mathbf{q})\mathbf{F} + [I_n - J^T(\mathbf{q})P^T(\mathbf{q})]\Gamma_o. \tag{24}$$

If, for any Γ_o, the end-effector is only subjected to $\mathbf{F}$, equation (24) yields

$$J(\mathbf{q})A^{-1}(\mathbf{q}) = [J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q})]P^T(\mathbf{q}); \tag{25}$$

which implies the identity between $P(\mathbf{q})$ and $\bar{J}(\mathbf{q})$.

Control of Redundant Manipulators

Similar to the case of non-redundant manipulators, the dynamic decoupling and control of the end-effector can be achieved by selecting an operational command vector of the form (9). The corresponding joint forces are

$$\Gamma = J^T(\mathbf{q})\Lambda_r(\mathbf{q})\mathbf{F}^*_m + \tilde{\mathbf{b}}_r(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}); \tag{26}$$

where $\tilde{\mathbf{b}}_r(\mathbf{q}, \dot{\mathbf{q}})$ is defined similarly to $\tilde{\mathbf{b}}(\mathbf{q}, \dot{\mathbf{q}})$.

Stability Analysis

Under the command vector (26), and with the assumption of a "perfect" compensation (or non-compensation) of the centrifugal and Coriolis forces, the manipulator is subjected to dissipative forces Γ_{dis} due to the velocity damping term $(-k_v\dot{\mathbf{x}})$ in $\mathbf{F}^*_m$, these forces are

$$\Gamma_{dis} = D(\mathbf{q})\dot{\mathbf{q}}; \tag{27}$$

with

$$D(\mathbf{q}) = -k_v J^T(\mathbf{q})\Lambda_r(\mathbf{q})J(\mathbf{q}). \tag{28}$$

$D(\mathbf{q})$ is an $n \times n$ negative semi-definite matrix of rank m. Although the manipulator is stable, since the condition

$$\dot{\mathbf{q}}^T D(\mathbf{q})\dot{\mathbf{q}} \le 0; \tag{29}$$

is satisfied, this redundant mechanism can still describe movements that are solutions of the equation

$$\dot{\mathbf{q}}^T D(\mathbf{q})\dot{\mathbf{q}} = 0. \tag{30}$$

An example of such a behavior is shown in Figure 2a. The end-effector of a simulated three-degree-of-freedom planar manipulator is controlled under (26). The end-effector goal position coincides with its current position, while the three joints are assumed to have initially non-zero velocities (0.5rad/s has been used).

Asymptotic stabilization of the system can be achieved by the addition of dissipative joint forces [Khatib 1980]. These forces

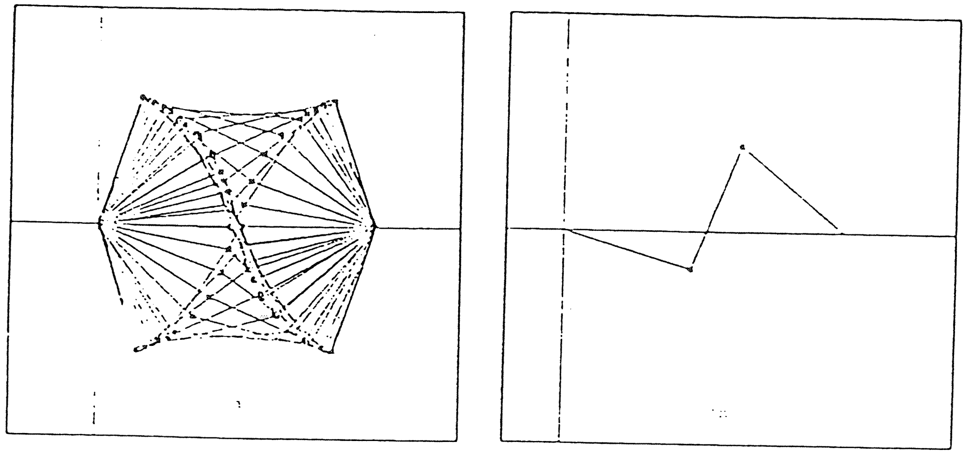

Figure 2: Stabilization of a Redundant Manipulator.

can be selected to act in the null space of the Jacobian matrix [Khatib 1985]. This precludes any effect of the additional forces on the end-effector and maintains its dynamic decoupling. Using (23) these additional stabilizing joint forces are of the form

$$\Gamma_{ns} = [I_n - J^T(q)\bar{J}^T(q)]\Gamma_s. \tag{31}$$

By selecting

$$\Gamma_s = -k_{vq}A(q)\dot{q}; \tag{32}$$

the vector Γ_{ns} becomes

$$\Gamma_{ns} = \Gamma_s + J^T(q)\Lambda_r(q)F_{rs}; \tag{33}$$

with

$$F_{rs} = k_{vq}\dot{x}. \tag{34}$$

Finally, the joint force command vector can be written as

$$\Gamma = J^T(q)\Lambda_r(q)(F^*_m + F_{rs}) + \Gamma_s + \tilde{b}_r(q,\dot{q}) + g(q). \tag{35}$$

Under this form, the evaluation of the explicit expression of the generalized inverse of the Jacobian matrix is avoided. The matrix $D(q)$ corresponding to the new expression for the dissipative joint forces Γ_{ds} in the command vector (35) becomes

$$D(q) = -[(k_v - k_{vq})J^T(q)\Lambda_r(q)J(q) + k_{vq}A(q)]. \tag{36}$$

Now, the matrix $D(q)$ is negative definite and the system is asymptotically stable. Figure 2b shows the effects of this stabilization on the previous example of a simulated three-degree-of-freedom manipulator.

Constrained Motion Control

The extension to redundant manipulators of the results obtained in the case of non redundancy is straightforward. The generalized joint forces command vector becomes

$$\Gamma = J_0^T(q)[\Lambda_{r0}(q)(\Omega F^*_m + \tilde{\Omega}F^*_s + F_{rs}) + \tilde{\Omega}F^*_a] + \Gamma_s + \tilde{b}_{r0}(q,\dot{q}) + g(q); \tag{37}$$

where $\Lambda_{r0}(q)$ and $\tilde{b}_{r0}(q,\dot{q})$ are defined with respect to the basic Jacobian matrix $J_0(q)$.

Conclusion

A methodology for the description of end-effector constrained motion tasks based on the construction of the generalized task specification matrix has been proposed. For such tasks where both motion and active force control are involved, a unified approach for end-effector dynamic control within the framework of the operational space formulation has been presented. The use of the generalized task specification matrix has provided a more efficient control structure for real-time implementations.

Also, the end-effector equations of motion for a redundant manipulator system have been established, and an operational space control system for end-effector dynamic decoupling and control has been designed. The expression of joint forces of the nullspace of the Jacobian matrix consistent with the end-effector dynamic behavior has been identified and used for the asymptotic stabilization of the redundant mechanism. The resulting control system avoids the explicit evaluation of any generalized inverse or pseudo-inverse of the Jacobian matrix. Joint constraints, collision avoidance, and control of manipulator postures can be naturally integrated in this framework of operational space control systems.

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