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FORCE-BASED MOTION CONTROL OF ROBOT MANIPULATORS

Abstract

The paper discusses some kinematic issues associated with motion control of robot manipulators. The discussion focuses on two different modes of motion control: position-based motion control (resolved motion-rate control), which uses the inverse of the linear kinematic model; and force-based motion control approach based on the relationship between end-effector forces and generalized joint forces. First, we identify in the case of redundant mechanisms the set of joint forces associated with the null space of the force transformation and establish the general expression for the relationship between end-effector and joint forces. The two modes of motion control are then compared, with respect to their integration within control structures for combined motion and active force control, their dynamic performance, and the technological requirements they impose.

1 Introduction

An important kinematic issue associated with motion control of robot mechanisms, is the *inverse kinematic problem* or more generally the task transformation problem. This problem is raised by the discrepancy between the world where tasks are specified and the world where motions are controlled.

Tasks are specified with respect to the robot's end-effector or manipulated object, while motions are typically controlled through the action of servo-controllers that effect the positions and velocities of the robot's joints. Finding the set of joint trajectories, inputs to the joint servo-controllers, that would produce the specified task is the central issue in the task transformation problem.

Obviously, the need for solutions of the inverse kinematic problem is not limited to the motion control problem. The inverse kinematic is needed in workspace analysis, design, simulations, and planning of robot motions. By its computational complexity, however, the inverse kinematic problem becomes more critical in real-time control implementations. This is, for instance, the case of tasks where the robot is called to accommodate motion that cannot be pre-planned or to make corrections generated by external sensory devices.

The wide use of position-based motion control is partly a natural result of the state-of-the-art in manipulator mechanical technology. Current manipulator technology relies almost exclusively on the concept of joint position control, whereas a prerequisite to force-based motion control implementation is the manipulator's ability to achieve precise control of joint torques. This ability, however, is considerably restricted by the nonlinearities and friction inherent in the actuator-transmission systems generally used in most industrial robots.

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These limitations have, in addition, a major impact on the dynamic performance that can be achieved. Despite the theoretical advances in manipulator control, PID controllers are still largely dominant in industrial robot systems. With PID controllers each joint is independently controlled. The dynamic interaction between joints is ignored, and the disturbance rejection of the dynamic forces relies on the use of large gains and high servo rates.

Robot joint torque control ability is essential not only for achieving higher dynamic performance, but also for the implementation of many force-based part mating operations. Active force control which has emerged as one of the basic means to extend robot capabilities also requires joint torque control capability.

In recent years, there has been an important effort to close the gap between the technologies in robot mechanisms and robot control. Recent trends and ongoing developments suggest that a new generation of force controlled robot systems is currently under development, *e.g.* Direct-Drive arms, the ARTISAN manipulator and micro-manipulator system (Roth et al. 1988). With the emergence of new capabilities for effective joint torque control, force-based motion control will become a natural control modality for robot manipulators.

2 Kinematic Control

The computation complexity of the inverse kinematic problem has led to solutions based on the inverse of the linearized kinematic model. This model expresses the relationship between the vector $\delta \mathrm{q}$ associated with the variations of joint positions and the vector $\delta \mathrm{x}$ associated with the corresponding variations of the positions and orientations of the end-effector,

$$\delta \mathrm{x} = J(\mathrm{q})\delta \mathrm{q}; \tag{1}$$

where $J(\mathrm{q})$ is the Jacobian matrix. For an n-degree-of-freedom manipulator with an end-effector operating in an m-dimensional space, $J(\mathrm{q})$ is an $n \times m$ matrix.

2.1 Position-Based Motion Control

Using the linearized kinematic model (1), Whitney (1972) proposed the *resolved motion-rate control* approach for the coordination of manipulator joint motions. The resolved motion-rate control uses the inverse of the linear relationship in equation (1). For a non-redundant manipulator, *i.e.* $n = m$, the solution is simply

$$\delta \mathrm{q} = J^{-1}(\mathrm{q})\delta \mathrm{x}. \tag{2}$$

For a given trajectory of the end-effector, motion control is achieved by continuously controlling the manipulator from the current position q to the position $\mathrm{q} + \delta \mathrm{q}$.

Redundant Manipulator Control The position and orientation of the end-effector of a redundant mechanism can be obtained with an infinite number of postures of its links. Generalized inverses and pseudo-inverses (Whitney 1972, Liegois 1977, Fournier 1980, Hanafusa et al. 1983) have been used to solve the kinematic equation (1). Using a generalized inverse $J^{\#}(\mathrm{q})$ of the Jacobian matrix, the general solution of the system (1) is

$$\delta \mathrm{q} = J^{\#}(\mathrm{q})\delta \mathrm{x} + [I - J^{\#}(\mathrm{q})J(\mathrm{q})]\delta \mathrm{q}_0; \tag{3}$$

where I is the identity matrix of appropriate dimensions and $\delta\mathrm{q}_0$ denotes an arbitrary vector. The matrix $[I - J^{\#}(\mathrm{q})J(\mathrm{q})]$ defines the null space associated with $J^{\#}(\mathrm{q})$, and vectors of the form $[I - J^{\#}(\mathrm{q})J(\mathrm{q})]\delta\mathrm{q}_0$ correspond to zero-variation of the position and orientation of the end-effector. The additional freedom of motion associated with null space is generally used to minimize some criteria.

2.2 Force-Based Motion Control

In the resolved motion-rate control approach, the inverse kinematic problem is replaced by a computationally less difficult problem which involves solving a system of linear equations. With the force-based motion control approach, the whole issue of task transformation is eliminated. End-effector motions are directly controlled by forces and moments acting along or about the directions where the task is described. These forces and moments are created by the application of a set of generalized joint forces. The basic relationship between end-effector forces and joint forces is given by

$$\Gamma = J^T(\mathrm{q})\mathrm{F}, \tag{4}$$

Redundant Manipulator Control In the case of redundant manipulators, the relationship between end-effector forces and joint forces of equation (4) becomes incomplete. At a given configuration of the mechanism, we have seen that there is an infinity of elementary displacements of the redundant mechanism that could take place without altering the configuration of end-effector. Those are the displacement in the null space associated with the generalized inverse of the Jacobian matrix.

With respect to forces, there is also an infinity of joint force vectors that can be applied without effecting the resulting forces reflected at the end-effector. Those are the joint forces acting along the directions of the null space.

Let us consider the virtual joint displacement

$$\delta\mathrm{q} = J^{\#}(\mathrm{q})\delta\mathrm{x} + [I - J^{\#}(\mathrm{q})J(\mathrm{q})]\delta\mathrm{q}_0.$$

The virtual work δW done by the generalized joint forces Γ in this virtual displacement is

$$\delta W = \delta W_1 + \delta W_2;$$

with

$$\delta W_1 = [J^{\#^T}(\mathrm{q})\Gamma]^T \delta\mathrm{x};$$

and

$$\delta W_2 = \{[I - J^{\#}(\mathrm{q})J(\mathrm{q})]^T\Gamma\}^T \delta\mathrm{q}_0.$$

δW_1 corresponds to the virtual work done by a vector F of end-effector forces in the virtual displacements $\delta\mathrm{x}$, and δW_2 is the virtual work done by the vector $[I - J^T(\mathrm{q})J^{\#^T}(\mathrm{q})]\Gamma$ of joint forces in the virtual displacement $\delta\mathrm{q}_0$. $[I - J^T(\mathrm{q})J^{\#^T}(\mathrm{q})]$ defines the null space of generalized joint forces associated with the generalized inverse $J^{\#}(\mathrm{q})$.

The general expression of the relationship between end-effector forces and generalized joint forces becomes

$$\Gamma = J^T(q)F + [I - J^T(q)J^{\#^T}(q)]\Gamma_0; \tag{5}$$

where Γ_0 is an arbitrary generalized joint force vector.

While the vector **F** will be used for the control of the end-effector, the joint torque vector Γ_0 will allow the control of the internal joint motions. This can be simply achieved by selecting Γ_0 as the gradient of a potential function which has its minimum at the desired manipulator's posture.

It should be noted that a further dynamic analysis would show that the generalized inverse involved in the previous relationship (equation 5) is not arbitrary. A generalized inverse that is consistent with the system's dynamics is shown to be unique (Khatib 1987). This is the generalized inverse corresponding to the solution that minimizes the manipulator's instantaneous kinetic energy. Furthermore, the particular structure for the dynamic control implementation allows to avoid the evaluation of this generalized inverse.

3 Summary and Discussion

The selection of an appropriate methodology for robot control must be based on the selection of the robot mechanical technology. If joint position controlled robots were to be used, non-dynamic position-based techniques would be the most effective. However, the capabilities of such systems, particularly in force control and part mating operations, is very limited.

For robot mechanisms where joint torque control capability is available, force-based motion control is clearly the most appropriate approach to be taken. In this approach, control forces can be easily designed to achieve the coordination of joint motions for many complex tasks (goal position, trajectory and surface tracking, collision and joint limit avoidance), whereas only elementary displacement of the end-effector are allowed in the linearized inverse kinematic model.

In part mating operations, the use of position-based motion control could result in two different command vectors for motion and active force control. For instance, both models (equations 2 and 4) are involved in the synthesis of the hybrid position/force control (Craig and Raibert 1981). With the force-based control approach, motion control and active force control are both achieved by the same command vector (Khatib 1987). This provides a unified approach based on the relationship given in equation (4). The unified approach to motion and force control has an important impact on the dynamic decoupling, stabilization, and control of end-effector motions and active forces.

Another important implication of the use of force-based motion control has been the development of various sensor-based strategies (Shashank and Khatib 1987) for precision assembly operations. These strategies has been used for the construction of various force-based compliant motion primitives for part mating tasks.

References

Fournier, A. 1980 (April). Génération de Mouvements en Robotique. Application des Inverses Généralisées et des Pseudo Inverses. *Thèse d'Etat*, Mention Science, Université des Sciences et Techniques du Languedoc, Montpellier, France.

Hanafusa, H., Yoshikawa, T., and Nakamura, Y., 1981. Analysis and Control of Articulated Robot Arms with Redundancy. *8th IFAC World Congress*, vol. XIV, pp. 38-83.

Khatib, O. 1987. A Unified Approach to Motion and Force Control of Robot Manipulators: The Operational Space Formulation. *IEEE J. on Robotics and Automation*, vol. 3, no. 1.

Liegois, A. 1977. Automatic Supervisory Control of the Configuration and Behavior of Multibody Mechanisms. *IEEE Trans. on Systems, Man and Cybernetics*, vol. 7, no. 12.

Raibert, M. and Craig, J. 1981 (June). Hybrid Position/Force Control of Manipulators. *ASME Journal of Dynamic Systems, Measurement, and Control*, pp. 126-133.

Roth, B. et al. 1988. The Design of the ARTISAN Research Manipulator System. in preparation.

Shekhar, S. and Khatib, O. 1987. "Force Strategies in Real-Time Fine Motion Assembly," ASME Winter Annual Meeting, Boston.

Whitney, D.E. 1972. The Mathematics of Coordinated Control of Prosthetic Arms and Manipulators. Journal of Dynamic Systems, Measurement, and Control, pp. 303-309.