The International Sym. of Robotics Research (ISRR), Santa Cruz, Aug., 1987

Object Manipulation in a Multi-Effector Robot System

Oussama Khatib

Robotics Laboratory
Computer Science Department
Stanford University

Abstract: This paper presents a framework for dealing with the problem of object manipulation in a system of multiple-robot manipulators. In this framework, the multi-effector/object system is treated as an augmented object representing the total masses and inertias perceived at some operational point actuated by the total effector forces acting at that point. This model is used for the dynamic decoupling, motion, and active force control of the system. The allocation of forces at the level of effectors is based on minimization of the total actuator joint force activities. The approach is extended to the case of redundant mechanisms.

1 Introduction

Object manipulation in multi-manipulator robot systems has recently received increased attention. (Alford and Belyeu, 1984) studied the coordination of two arms. Their control system is organized in a master/slave fashion, and a motion coordination procedure is used to minimize the error in the relative position between the two manipulator effectors. (Zheng and Luh, 1986) have treated the control problem of two manipulators as a "leader" and a "follower" system. The joint torques of the follower are obtained directly from the constraint relationships between the two manipulators allowing a coordinated control of the system.

The problem of motion and force control of multiple manipulators has been investigated in (Hayati, 1986). In his proposed approach, the load is partitioned among the arms. Dynamic decoupling and motion control are then achieved at the level of individual manipulator effectors. In the force control subspace, the magnitude of forces is minimized.

(Tarn, Bejczy, and Yun; 1987) developed the closed chain dynamic model of a two-manipulator system with respect to a selected set of generalized joint coordinates. Nonlinear feedback and output decoupling techniques were then used to linearize and control the system in task coordinates.

Joint space dynamic models only provide a description of the interaction between joint motions. The control of object motion and active forces requires the description of how motions along different axes are interacting, and how the apparent or equivalent inertia or mass of the object varies with configurations and directions. In this paper, the equations of motion of a multi-effector/object system are established and the unified approach for motion and active force control (Khatib, 1987) is extended to the control of this type of robot system. This approach is also extended to the case of redunadnt mechanisms.

2 Outline of the Approach

Manipulator joints have been generally treated as motion generator devices. From this perspective, the control of the effector motion of a single manipulator is viewed as a joint motion coordination problem. In a multi-arm system, the problem of object manipulation has been formulated similarly, *i.e.* coordination of the motions of the individual arms or the individual effectors.

In the operational space framework, the control of the end-effector is based on the selection of the operational forces generated at the end-effector as a command vector. The relationships between these forces and the effector inertial and gravitional forces are used to achieve dynamic decoupling. The operational forces are produced by submitting the manipulator to the corresponding joint forces. In this approach, the involvement in the control structure of the manipulator joints is limited to the generation of joint forces.

Similarly, in a multi-effector system, the manipulators are viewed as the mechanical support for the effectors to provide forces and moments at the level of the manipulated object. If the dynamics of these effectors were negligible, the equations of motion of the system would be given by the relationship between the inertial and gravitional characteristics of the manipulated object and the total effector forces acting on it. An effector, however, is not a pure generator of forces. In motion, the effector is submitted to significant inertial forces. These inertial forces are

Published in Robotics Research 4, Cambridge: MIT Press, 1988

given, in the case of a single effector, by the operational space dynamic model.

A multi-effector/object system will be treated as an *augmented object* representing the total masses and inertias perceived at some operational point. This object is submitted to the vector of total force resulting from the combined action of the effectors at that point. The dynamic behavior of this system is described by the relationship between the vector of total force, *i.e.* the command vector, and the inertial and gravitional characteristics of the augmented object.

This model is then used to achieve dynamic decoupling and control of the system. The realization of the command vector is obtained by partitioning it into a set of collinear vectors allocated to the various effectors. The allocation of forces is based on the minimization of the total joint actuator activities.

3 Single Effector/Object System

In this section, the framework of operational space dynamics and control is summarized and the effect of the manipulated object on the effector dynamics is described.

3.1 Effector Operational Point

The position and orientation, with respect to a reference coordinate frame $\mathcal{R}_r(\mathcal{O}_r, \mathbf{x}_r, \mathbf{y}_r, \mathbf{z}_r)$, of the effector is described by the relationship between $\mathcal{R}_r$ and a coordinate frame $\mathcal{R}_e(\mathcal{O}_e, \mathbf{x}_e, \mathbf{y}_e, \mathbf{z}_e)$ attached to this effector. The effector position is given by the coordinate in $\mathcal{R}_r$ of the point $\mathcal{O}_e$, and its orientation is defined by the rotation transformation of $\mathcal{R}_e$ with respect to $\mathcal{R}_r$. The selection of the location on the effector of the point $\mathcal{O}_e$ (*e.g.* effector center of mass, tip) will depend on the type of operation to be performed and the way this operation is specified.

$\mathcal{O}_e$ is called the *effector operational point*. It is with respect to $\mathcal{O}_e$ that translational and rotational motions and active forces of the effector are specified.

3.2 Effector Equations of Motion

The number of independent parameters required to completely specify, in $\mathcal{R}_r$, the effector configuration is defined as the number of *effector degrees of freedom*. Various representations for the position (*e.g.* Cartesian, cylindrical, or spherical) and orientation (*e.g.* Euler angles, Euler parameters, direction cosines) can be found.

An *operational coordinate system* associated with an m-degree-of-freedom effector and a point $\mathcal{O}_e$, is a set $\mathbf{x}$ of m *independent* parameters describing the effector position and orientation in a frame of reference $\mathcal{R}_r$. For a nonredundant n-degree-of-freedom manipulator, *i.e.* $n = m$, these parameters form a set of configuration parameters in a domain $\mathcal{D}$ of the operational space (Khatib, 1987) and constitute, therefore, a system of *generalized coordinates*. The kinetic energy of the holonomic articulated mechanism is a quadratic form of the generalized operational velocities

$$T(\mathbf{x}, \dot{\mathbf{x}}) = \frac{1}{2}\dot{\mathbf{x}}^T \Lambda(\mathbf{x})\dot{\mathbf{x}}; \tag{1}$$

where $\Lambda(\mathbf{x})$ designates the $m \times m$ symmetric matrix of the quadratic form, *i.e.* the kinetic energy matrix. The kinetic energy can be similarly expressed with respect to other systems of generalized coordinates. Using the Jacobian matrix that relates the two systems of generalized velocities, the relationship between kinetic energy matrices associated with different generalized coordinates can be established by exploiting the identity between the two expressions of kinetic energy. With $A(\mathbf{q})$ being the kinetic energy matrix associated with the system $\mathbf{q}$ of generalized joint coordinates, and $J(\mathbf{q})$ the Jacobian matrix associated with the generalized operational velocities $\dot{\mathbf{x}}$, the matrix associated with the operational coordinates $\mathbf{x}$ is

$$\Lambda(\mathbf{x}) = J^{-T}(\mathbf{q})A(\mathbf{q})J^{-1}(\mathbf{q}). \tag{2}$$

Let $\mathbf{F}$ be the vector of generalized operational forces associated with the generalized coordinates $\mathbf{x}$. Using the Lagrangian formalism, the end-effector equations of motion are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{x}}}\right) - \frac{\partial L}{\partial \mathbf{x}} = \mathbf{F}; \tag{3}$$

where the Lagrangian $L(\mathbf{x}, \dot{\mathbf{x}})$ is

$$L(\mathbf{x}, \dot{\mathbf{x}}) = T(\mathbf{x}, \dot{\mathbf{x}}) - U(\mathbf{x}); \tag{4}$$

and $U(\mathbf{x})$ represents the potential energy due to gravity. Let $\mathbf{p}(\mathbf{x})$ be the vector of gravity forces

$$\mathbf{p}(\mathbf{x}) = \nabla U(\mathbf{x}). \tag{5}$$

The effector equations of motion in operational space are given by (Khatib, 1980 and 1987)

$$\Lambda(\mathbf{x})\ddot{\mathbf{x}} + \Pi(\mathbf{x})[\dot{\mathbf{x}}\dot{\mathbf{x}}] + \mathbf{p}(\mathbf{x}) = \mathbf{F}; \tag{6}$$

$\Pi(\mathbf{x})$ represents the $m \times m(m+1)/2$ matrix of centrifugal and Coriolis forces. The elements of the matrix $\Pi(\mathbf{x})$ can be obtained from the Christoffel symbols $\pi_{i\,jk}$ given as a function of the partial derivatives of $\Lambda(\mathbf{x})$ with respect to the generalized coordinates $\mathbf{x}$,

$$\pi_{i,jk} = \frac{1}{2}\left(\frac{\partial \lambda_{ij}}{\partial x_k} + \frac{\partial \lambda_{ik}}{\partial x_j} - \frac{\partial \lambda_{jk}}{\partial x_i}\right). \tag{7}$$

The matrix of centrifugal and Coriolis forces is given by
$\Pi(\mathbf{x}) =$

$$\begin{bmatrix} \pi_{1,11} & \pi_{1,12} & \dots & \pi_{1,1m} & \pi_{1,22} & \dots & \pi_{1,mm} \\ \pi_{2,11} & \pi_{2,12} & \dots & \pi_{2,1m} & \pi_{2,22} & \dots & \pi_{2,mm} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \pi_{m,11} & \pi_{m,12} & \dots & \pi_{m,1m} & \pi_{m,22} & \dots & \pi_{m,mm} \end{bmatrix}. \tag{8}$$

$[\dot{\mathbf{x}}\dot{\mathbf{x}}]$ represents the symbolic notation of the $m(m+1)/2 \times 1$ column matrix

$$[\dot{\mathbf{x}}\dot{\mathbf{x}}] = [\dot{x}_1^2 \;\; 2\dot{x}_1\dot{x}_2 \dots 2\dot{x}_1\dot{x}_m \;\; \dot{x}_2^2 \dots \dot{x}_m^2]^T \tag{9}$$

3.3 Effect of a Load

The kinetic energy matrix $\Lambda(\mathbf{x})$ associated with the operational coordinates $\mathbf{x}$ describes the inertial characteristics of the effector as perceived at the point $\mathcal{O}_e$. The addition of a load will result in an increase in the total kinetic energy.
Let m_l and I_l be the mass and inertia matrix of the load with respect to $\mathcal{R}_e$. The additional kinetic energy due to the load is

$$T_{\text{load}} = \frac{1}{2}[m_l \mathbf{v}^T \mathbf{v} + \omega^T I_l \omega]; \tag{10}$$

where $\mathbf{v}$ and ω are the vectors of linear and angular velocities.
The generalized operational velocities $\dot{\mathbf{x}}$ are related to the linear and angular velocities by a matrix $E(\mathbf{x})$ expressed as function of the operational coordinates $\mathbf{x}$,

$$\dot{\mathbf{x}} = E(\mathbf{x}) \begin{bmatrix} \mathbf{v} \\ \omega \end{bmatrix}. \tag{11}$$

The matrix $E(\mathbf{x})$ is dependent on the type of coordinates selected to represent the position and orientation of the effector. By introducing the column matrices $\mathbf{x}_p$ and $\mathbf{x}_r$ defining, respectively, the selected coordinates for the position and orientation,

$$\mathbf{x}(\mathbf{q}) = \begin{bmatrix} \mathbf{x}_p \\ \mathbf{x}_r \end{bmatrix}; \tag{12}$$

$E(\mathbf{x})$ can be written as

$$E(\mathbf{x}) = \begin{bmatrix} E_p(\mathbf{x}_p) & 0 \\ 0 & E_r(\mathbf{x}_r) \end{bmatrix}. \tag{13}$$

Using equation (11), the kinetic energy due to the load can be written in the form

$$T_{\text{load}} = \frac{1}{2}\dot{\mathbf{x}}^T \Lambda_l(\mathbf{x})\dot{\mathbf{x}}; \tag{14}$$

where the matrix of kinetic energy with respect to $\mathbf{x}$ is

$$\Lambda_l(\mathbf{x}) = E^{-T}(\mathbf{x}) M_l E^{-1}(\mathbf{x};) \tag{15}$$

with

$$M_l = \begin{bmatrix} m_l \mathbf{1} & \mathbf{0} \\ \mathbf{0} & I_l \end{bmatrix}; \tag{16}$$

where $\mathbf{1}$ and $\mathbf{0}$ are the unit and zero matrices of appropriate dimension.
For instance, for a selection of Cartesian coordinates and Euler angles, *i.e.* $(x, y, z, \psi, \theta, \phi)$, the matrix $E^{-1}(\mathbf{x})$ in equation (15) is

$$E^{-1}(\mathbf{x}) = \begin{bmatrix} \mathbf{1} & 0 \\ 0 & E_r^{-1}(\mathbf{x}_r) \end{bmatrix};$$

where

$$E_r^{-1}(\mathbf{x}_r) = \begin{bmatrix} 0 & \cos\psi & \sin\psi\sin\theta \\ 0 & \sin\psi & -\cos\psi\sin\theta \\ 1 & 0 & \cos\theta \end{bmatrix}.$$

Lemma 1

The kinetic energy matrix of the effector and load system is the matrix

$$\Lambda_{\text{effector+load}}(\mathbf{x}) = \Lambda_{\text{effector}}(\mathbf{x}) + \Lambda_{\text{load}}(\mathbf{x}). \tag{17}$$

This is a straightforward implication of the evaluation, with respect to the operational coordinates, of the total kinetic energy of the system.

3.4 Effector/Object Equations of Motion

The equations of motion of the effector and load system become

$$\Lambda_{e+l}(\mathbf{x})\ddot{\mathbf{x}} + \Pi_{e+l}(\mathbf{x})[\dot{\mathbf{x}}\dot{\mathbf{x}}] + \mathbf{p}_{e+l}(\mathbf{x}) = \mathbf{F}; \tag{18}$$

where, using equations (7), (8), and (15), $\Pi_{e+l}(\mathbf{x})$ is given by

$$\Pi_{e+l}(\mathbf{x}) = \Pi_e(\mathbf{x}) + \Pi_l(\mathbf{x}). \tag{19}$$

$\Pi_l(\mathbf{x})$ is the $m \times m(m+1)/2$ matrix of centrifugal and Coriolis forces associated with the load and obtained from the partial derivatives of $\Lambda_l(\mathbf{x})$. Also, the resulting gravity vector can be written as

$$\mathbf{p}_{e+l}(\mathbf{x}) = \mathbf{p}_e(\mathbf{x}) + \mathbf{p}_l(\mathbf{x}); \tag{20}$$

where $\mathbf{p}_l(\mathbf{x})$ is the gravity vector obtained from the potential energy $U_l(\mathbf{x})$ associated with the load as in equation (5).

4 Multi-Effector/Object System

Let us consider the problem of manipulating an object with a system of N robot manipulators. The effectors of each of these manipulators are assumed to have the same number of degrees of freedom, m, and to be rigidly connected to the manipulated object. Let $\mathcal{O}_o$ be the selected operational point attached to this object. This point is fixed with respect to each of the effectors.
Let $\Lambda_l(\mathbf{x}_o)$ be the kinetic energy matrix associated with the load computed as in equation (15) and expressed with respect to $\mathcal{O}_o$ and the operational coordinates $\mathbf{x}_o$. Let $\Lambda_i(\mathbf{x}_o)$ be the kinetic energy matrix associated with the i^{th} effector.

Lemma 2

The kinetic energy matrix of the N-effector/object system is

$$\Lambda_s(\mathbf{x}_o) = \Lambda_l(\mathbf{x}_o) + \sum_{i=1}^{N} \Lambda_i(\mathbf{x}_o). \tag{21}$$

This is simply a generalization of Lemma 1. Equation (21) is obtained by evaluating the total kinetic energy of the N effectors and object system expressed with respect to the operational velocities,

$$T = \frac{1}{2}\dot{\mathbf{x}}_o^T \Lambda_l(\mathbf{x}_o)\dot{\mathbf{x}}_o + \sum_{i=1}^{N} \frac{1}{2}\dot{\mathbf{x}}_o^T \Lambda_i(\mathbf{x}_o)\dot{\mathbf{x}}_o.$$

4.1 Multi-Effector/Object Equations of Motion

The system considered here is the system resulting from rigidly connecting an object, to the effectors of N n-degree-of-freedom manipulators. This system is formed by $N(n-1)+1$ links, and one ground link connected through Nn one-degree-of-freedom joints. The number n_s of degrees of freedom of this system is given by the difference between the number of total degrees of freedom of these links obtained before the connection and the number of total degrees of freedom lost by the joint constraints after the connection. This number is given by the Grübler formula (Hartenberg and Denavit, 1964),

$$n_s = n_0(n_{\text{link}} - 1) - (n_0 - 1)n_{\text{joint}};$$

where n_{link} and n_{joint} are the numbers of total links and joints and n_0 is the number of degrees of freedom of an unconnected link (3 in the planar case and 6 in the spatial case). For the system of N n-degree-of-freedom manipulators and object considered here,

$$n_s = n_0[N(n-1)+1] - (n_0 - 1)Nn. \tag{22}$$

With the assumption of non redundancy, the number of degrees of freedom in the planar case ($n_0 = n = m = 3$) is $n_s = 3$. This number is $n_s = 6$ in the spatial case ($n_0 = n = m = 6$).
The number of operational coordinates, m, is equal to the number of degrees of freedom, n_s, of the mechanism. These coordinates form, therefore, a set of generalized coordinates for the system.
The kinetic energy matrix of the system expressed with respect to the generalized operational coordinates $\mathbf{x}_o$ is given by equation (21). The equations of motion of the multi-effector/object system are

$$\Lambda_s(\mathbf{x}_o)\ddot{\mathbf{x}}_o + \Pi_s(\mathbf{x}_o)[\dot{\mathbf{x}}_o\dot{\mathbf{x}}_o] + \mathbf{p}_s(\mathbf{x}_o) = \mathbf{F}_o; \tag{23}$$

where the matrix, $\Pi_s(\mathbf{x}_o)$, of centrifugal and Coriolis forces is obtained using equations (7), (8), and (15)

$$\Pi_s(\mathbf{x}_o) = \Pi_l(\mathbf{x}_o) + \sum_{i=1}^{N} \Pi_i(\mathbf{x}_o); \tag{24}$$

where $\Pi_l(\mathbf{x}_o)$ and $\Pi_i(\mathbf{x}_o)$ are the $m \times m(m+1)/2$ matrix of centrifugal and Coriolis forces associated with $\Lambda_l(\mathbf{x}_o)$ and $\Lambda_i(\mathbf{x}_o)$ respectively. The gravity vector is

$$\mathbf{p}_s(\mathbf{x}_o) = \mathbf{p}_l(\mathbf{x}_o) + \sum_{i=1}^{N} \mathbf{p}_i(\mathbf{x}_o); \tag{25}$$

where $\mathbf{p}_l(\mathbf{x}_o)$ and $\mathbf{p}_i(\mathbf{x}_o)$ are the gravity vectors associated with the object and the i^{th} effector.

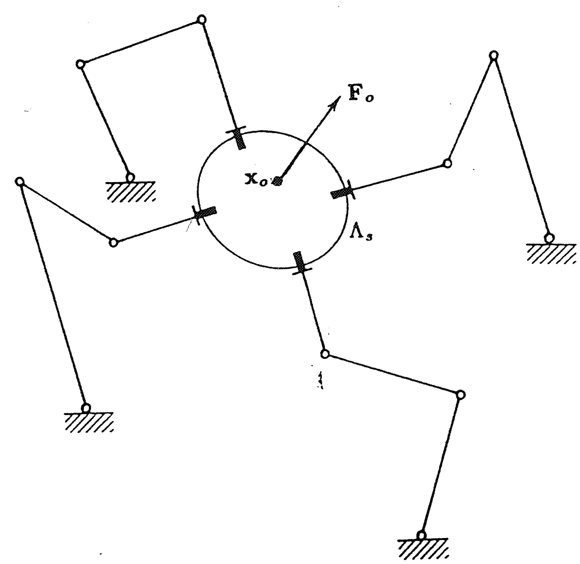

Figure 1: A Multi-Effector/Object System

4.2 Augmented Object Control

The equations of motion (23) establish the relationships between the positions, velocities, and accelerations of the multi-effector/load system and the operational forces acting on it, as illustrated in Figure 1. These equations can be viewed as describing the motion of an *augmented object* submitted to the operational forces $\mathbf{F}_o$, created by a set of effectors acting (as m-dimensional actuators) at the operational point $\mathcal{O}_o$.

The control of this object in operational space is based on the selection of $\mathbf{F}_o$ as a command vector. These generalized operational forces are the resultants of the forces produced by each of the N effectors at the operational point $\mathcal{O}_o$.

$$\mathbf{F}_o = \sum_{i=1}^{N} \mathbf{F}_{o_i}. \tag{26}$$

The effector's operational forces $\mathbf{F}_{o_i}$ are generated by the corresponding manipulator actuators. The generalized joint force vector $\mathbf{\Gamma}_i$ corresponding to $\mathbf{F}_{o_i}$ is given by

$$\mathbf{\Gamma}_i = J_{o_i}^T(\mathbf{q}_i)\ \mathbf{F}_{o_i}; \tag{27}$$

where $\mathbf{q}_i$ and $J_{o_i}^T(\mathbf{q}_i)$ are, respectively, the vector of joint coordinates and the Jacobian matrix computed with respect to $\mathbf{x}_o$ and associated with the i^{th} manipulator.

The dynamic decoupling and motion control of the object in operational space is achieved by selecting the control structure

$$\mathbf{F}_o = \widehat{\Lambda}_s(\mathbf{x}_o)\mathbf{F}_o^* + \widehat{\Pi}_s(\mathbf{x}_o)[\dot{\mathbf{x}}_o\dot{\mathbf{x}}_o] + \widehat{\mathbf{p}}_s(\mathbf{x}_o); \tag{28}$$

where, $\widehat{\Lambda}_s(\mathbf{x}_s)$, $\widehat{\Pi}_s(\mathbf{x}_o)$, and $\widehat{\mathbf{p}}_s(\mathbf{x}_o)$ represent the estimates of $\Lambda_s(\mathbf{x}_o)$, $\Pi_s(\mathbf{x}_o)$, and $\mathbf{p}_s(\mathbf{x}_o)$. $\mathbf{F}_o^*$ represents the input of the decoupled system. With perfect nonlinear dynamic decoupling, the system (23) becomes equivalent to a *single unit mass*, I_m, moving in the m-dimensional space,

$$I_m\ddot{\mathbf{x}}_o = F_o^*; \tag{29}$$

The unified operational command vector for motion and force control can be obtained similarly to the case of a single manipulator (Khatib and Burdick, 1986; Khatib, 1987). This vector will involve in addition the generalized task specification matrices.

4.3 Allocation of Effector Forces

The control structure (28) provides the net force $\mathbf{F}_o$ to be applied to the augmented object at $\mathcal{O}_o$. The criterion for distributing this force between effectors will be based on the minimization of total actuator activities.

The force vector, $\mathbf{F}_{o_i}$, to be produced by the i^{th} effector should be aligned with $\mathbf{F}_o$ and acting in the same direction,

$$\mathbf{F}_{o_i} = \alpha_i\mathbf{F}_o; \quad \text{with } \alpha_i > 0. \tag{30}$$

In addition, the set of N positive numbers α_i must satisfy

$$\sum_{i=1}^{N}\alpha_i = 1. \tag{31}$$

The actuator joint forces required by the i^{th} manipulator is

$$\mathbf{\Gamma}_i = \alpha_i J_{o_i}^T(\mathbf{q}_i)\ \mathbf{F}_o; \tag{32}$$

The problem now is to find the set of N positive numbers, α_1, α_2,...,α_N such that the overall effort of the actuators is minimized.

Let us consider the vector of joint forces τ_i corresponding to the total operational forces $\mathbf{F}_o$

$$\tau_i = J_{o_i}^T(\mathbf{q}_i)\ \mathbf{F}_o; \tag{33}$$

τ_i represents the actuator joint forces that would be assigned to the i^{th} manipulator, if this manipulator alone were to produce the total operational force $\mathbf{F}_o$. Let τ_{ij} be the j^{th} component of τ_i. Actuator joint forces are limited. Let $\overline{\gamma}_{ij}$ be the magnitude of the maximal bounds on the j^{th} actuator force of the i^{th} manipulator.

The number $|\tau_{ij}|/\overline{\gamma}_{ij}$ represents a measure of the effort that will be required by the j^{th} actuator if the i^{th} manipulator alone produced the total operational forces $\mathbf{F}_o$. The effort of the i^{th} manipulator can be characterized by

$$r_i = \max_j\{|\tau_{ij}|/\overline{\gamma}_{ij}\}; \tag{34}$$

which corresponds to the greatest effort. r_i is a positive number, which would be greater than one if the requested joint forces cannot be achieved by the i^{th} manipulator alone.

In order to minimize the overall effort, the weighting numbers α_1, α_2, ..., and α_N will be selected so that the effort is equally distributed, that is

$$\alpha_1 r_1 = \alpha_2 r_2 = \ldots = \alpha_N r_N. \tag{35}$$

Using equation (31), this corresponds to the solution

$$\alpha_i = \frac{\beta_i}{\beta_1 + \beta_2 + \ldots + \beta_N}; \tag{36}$$

where

$$\beta_i = \frac{r_1.r_2\ldots r_N}{r_i}. \tag{37}$$

5 Redundant Systems

When redundant manipulators are involved, the number of degrees of freedom of the entire system might increase. Its configuration then cannot be specified by a set of parameters that only describes the object position and orientation. Therefore, the dynamic behavior of the entire system cannot be described by a dynamic model in operational coordinates. The dynamic behavior of the augmented object itself, nevertheless, can still be described, and its equations of motion in operational space can still be established.
The number of degrees of freedom of the entire system, n_s, is given by (22). The number of degrees of redundacy of this system can be defined by $n_s - m_o$, where m_o is the number of degrees of freedom of the augmented object. It is important here to note that the system resulting from the connection of a set of individually redundant manipulators is not always redundant.
The freedom of the object is restricted by the freedom of the effectors. Let m_i be the number of degrees of freedom of the i^{th} effector before connection to the object. Constrained by the effectors, the object's number of degrees of freedom is

$$m_o \le \min_i \{m_i\}. \tag{38}$$

The inequality in (38) reflects the fact that additional constraints can be introduced by the connection of effectors. Connected to the object, all effectors will have the same number of degrees of freedom, m_o.
In order to be able to arbitrarily specify the position and orientation of the manipulated object, m_o must be equal to three in the planar case and six in the spatial case. If n_i represents the number of degrees of freedom of the i^{th} manipulator, the degree of redundancy of this manipulator is given by $n_i - m_o$.

5.1 Equations of Motion of a Single Manipulator

Before treating the case of a multi-effector/object system, we summarize the results in the case of a single redundant manipulator.

5.1.1 Joint Space Equations of Motion

The equations of motion of a single manipulator in joint space can be written in the form

$$A(\mathbf{q})\ddot{\mathbf{q}} + B(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \mathbf{g}(\mathbf{q}) = \mathbf{\Gamma}; \tag{39}$$

where $A(\mathbf{q})$ is the $n \times n$ joint space kinetic energy matrix, $B(\mathbf{q})$ is the $n \times n(n+1)/2$ matrix of centrifugal and Coriolis forces. $\mathbf{g}(\mathbf{q})$, and $\mathbf{\Gamma}$ represent, respectively, the gravity, and generalized forces in joint space.

5.1.2 Operational Space Equations of Motion

The dynamic behavior of the effector for a single redundant manipulator in operational space (Khatib, 1987) can be described by

$$\Lambda_r(\mathbf{q})\ddot{\mathbf{x}} + \Pi_r(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \mathbf{p}_r(\mathbf{q}) = \mathbf{F}; \tag{40}$$

where

$$\begin{aligned} \Lambda_r(\mathbf{q}) &= [J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q})]^{-1}; \\ \Pi_r(\mathbf{q}) &= \bar{J}^T(\mathbf{q})B(\mathbf{q}) - \Lambda_r(\mathbf{q})H(\mathbf{q}); \\ \mathbf{p}_r(\mathbf{q}) &= \bar{J}^T(\mathbf{q})\mathbf{g}(\mathbf{q}); \end{aligned} \tag{41}$$

and

$$\bar{J}(\mathbf{q}) = A^{-1}(\mathbf{q})J^T(\mathbf{q})\Lambda_r(\mathbf{q}). \tag{42}$$

The matrix $H(\mathbf{q})$ is defined by

$$H(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] = \dot{J}(\mathbf{q})\dot{\mathbf{q}}. \tag{43}$$

$\bar{J}(\mathbf{q})$ is actually a generalized inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's instantaneous kinetic energy.
The $m \times m$ matrix $\Lambda_r(\mathbf{q})$ is defined as a *pseudo-kinetic energy matrix*. $\Pi_r(\mathbf{q})$ represents the matrix of Centrifugal and Coriolis forces acting on the end-effector, and $\mathbf{p}_r(\mathbf{q})$ the gravity force vector.

5.2 Augmented Object in a Redundant Mechanism

The dynamic behavior of each of the effectors is described by an equation of the form (40), the dynamic behavior of the augmented object system is

$$\Lambda_{s_r}(\mathbf{q})\ddot{\mathbf{x}}_o + \Pi_{s_r}(\mathbf{q})\mathbf{v}(\dot{\mathbf{x}}_o, \dot{\mathbf{q}}) + \mathbf{p}_{s_r}(\mathbf{q}) = \mathbf{F}_o; \tag{44}$$

where

$$\begin{aligned} \Lambda_{s_r}(\mathbf{q}) &= \Lambda_l(\mathbf{x}_o) + \sum_{i=1}^{N} \Lambda_{r_i}(\mathbf{q}_i); \\ \Pi_{s_r}(\mathbf{q}) &= [\Pi_l(\mathbf{x}_o)|\Pi_{1_r}(\mathbf{q}_1)|\dots|\Pi_{N_r}(\mathbf{q}_N)]; \\ \mathbf{p}_{s_r}(\mathbf{q}) &= \mathbf{p}_l(\mathbf{x}_o) + \sum_{i=1}^{N} \mathbf{p}_{i_r}(\mathbf{q}_i); \end{aligned} \tag{45}$$

with

$$\begin{aligned} \mathbf{v}(\dot{\mathbf{x}}_o, \dot{\mathbf{q}}) &= [[\dot{\mathbf{x}}_o\dot{\mathbf{x}}_o]^T|[\dot{\mathbf{q}}_1\dot{\mathbf{q}}_1]^T|\dots[\dot{\mathbf{q}}_N\dot{\mathbf{q}}_N]^T]^T; \\ \mathbf{q} &= [\mathbf{q}_1^T\mathbf{q}_2^T\dots\mathbf{q}_N^T]^T; \end{aligned} \tag{46}$$

and

$$\mathbf{F}_o = \sum_{i=1}^{N} \mathbf{F}_{o_i}. \tag{47}$$

$\mathbf{F}_{o_i}$ is the operational force generated by the i^{th} effector.

5.3 Augmented Object Control: Redundant Case

As in the case of a nonredundant system, the dynamic decoupling and control of the multi-effector/object system can be achieved by selecting an operational command vector of the form of (28), that is

$$\mathbf{F}_o = \widehat{\Lambda}_{s_r}(\mathbf{q})\mathbf{F}_o^* + \widehat{\Pi}_{s_r}(\mathbf{q})\mathbf{v}(\dot{\mathbf{x}}_o\dot{\mathbf{q}}) + \widehat{\mathbf{p}}_{s_r}(\mathbf{q}_o). \quad (48)$$

The control vector $\mathbf{F}_o$ will be distributed between the effectors as in (30), and (31)

$$\mathbf{F}_{o_i} = \alpha_i \mathbf{F}_o; \quad \text{with} \quad \sum_{i=1}^{N} \alpha_i = 1;$$

where α_i is selected to minimize the overall actuator activities as in equations (36), and (37).

5.4 Stability of the Redundant Mechanism

As in the case of a single redundant manipulator (Khatib, 1987), the stability analysis of the multi-effector/object system shows that additional stabilizing joint forces and additional gravity compensations are required to achieve asymptotic stabilization of this system.
These joint forces must be selected appropriately in order to preclude any effect of the additional forces on the multi-effector/object system and to maintain its dynamic decoupling. This can be achieved by selecting these forces from the null space of the Jacobian transpose matrix associated with each manipulator.

5.4.1 Joint Forces in a Single Manipulator

Joint forces in a single manipulator system can be decomposed into

$$\Gamma = J^T(\mathbf{q})\mathbf{F} + [\mathbf{1}_n - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\Gamma_o; \quad (49)$$

where $\mathbf{1}_n$ is the $n \times n$ identity matrix. $\bar{J}(\mathbf{q})$, the generalized inverse that is consistent with the equations of motion of the manipulator and its effector, is given by

$$\bar{J}(\mathbf{q}) = A^{-1}(\mathbf{q})J^T(\mathbf{q})[J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q})]^{-1}. \quad (50)$$

Γ_o is an arbitrary joint force vector. Joint forces of the form $[\mathbf{1}_n - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\Gamma_o$ correspond to a null operational vector.
The generalized inverse given in equation (50) is consistent with the manipulator dynamic equations and is unique (Khatib, 1987). This generalized inverse is obtained as a function of the manipulator kinetic energy matrix $A(\mathbf{q})$. The determination of the generalized inverse associated with a manipulator involved in the multi-effector/object system will require the evaluation of the inertial characteristics reflected at that manipulator.
First let us examine how the joint space kinetic energy matrix in the case of a single manipulator is affected by the addition of a load.

5.4.2 Effect of a Load on a Single Manipulator

The addition of a load to the effector of a single manipulator will result in an increase in the kinetic energy of system. Let $\Lambda_{\text{load}}(\mathbf{x}_l)$ be the kinetic energy matrix associated with the load and expressed with respect to $\mathbf{x}_o$.

Lemma 3

The joint space kinetic energy matrix of a manipulator with load is the matrix

$$A_{\text{arm+load}}(\mathbf{q}) = A_{\text{arm}}(\mathbf{q}) + [J_l^T(\mathbf{q})\Lambda_{\text{load}}(\mathbf{x}_l)J_l(\mathbf{q})]. \quad (51)$$

Replacing the operational velocities by their expressions in terms of joint velocities, the total kinetic energy is

$$T = \frac{1}{2}\dot{\mathbf{q}}^T[A(\mathbf{q}) + J_l^T(\mathbf{q})\Lambda_l(\mathbf{x}_l)J_l(\mathbf{q})]\dot{\mathbf{q}}.$$

5.4.3 Reflected Load

The pseudo kinetic energy matrix $\Lambda_{s_r}(\mathbf{q})$ describes the inertial characteristics of the N-effector/object system as reflected at the operational point $\mathcal{O}_o$. Viewed from a given manipulator, the object and the other effectors can be seen as a load attached at the point $\mathcal{O}_o$ on its effector. The additional load perceived by the i^{th} manipulator is $\Lambda_{s_r}(\mathbf{q}) - \Lambda_{i_r}(\mathbf{q}_i)$. Following Lemma 3, the kinetic energy matrix of the manipulator resulting from this additional load is

$$A_{+_i}(\mathbf{q}) = A_i(\mathbf{q}_i) + J_i^T(\mathbf{q}_i)[\Lambda_{s_r}(\mathbf{q}) - \Lambda_{i_r}(\mathbf{q}_i)]J_i(\mathbf{q}_i). \quad (52)$$

The generalized inverse associated with the i^{th} manipulator and consistent with the dynamic behavior of the multi-effector/object system is given by

$$\bar{J}_i(\mathbf{q}) = A_{+_i}^{-1}(\mathbf{q})J_i^T(\mathbf{q}_i)[J_i(\mathbf{q}_i)A_{+_i}^{-1}(\mathbf{q})J_i^T(\mathbf{q}_i)]^{-1}. \quad (53)$$

Finally, the i^{th} manipulator joint forces are

$$\Gamma_i = \alpha_i J_i^T(\mathbf{q}_i)\mathbf{F}_o + [\mathbf{1}_n - J_i^T(\mathbf{q}_i)\bar{J}_i^T(\mathbf{q})]\Gamma_{i_o}; \quad (54)$$

where Γ_{i_o} is an arbitrary joint force vector. Asymptotic stabilization of the redundant system can be achieved by the addition of dissipative joint forces Γ_{i_s}. With the gravity compensation, the vector Γ_{i_o} is

$$\Gamma_{i_o} = \Gamma_{i_s} + g_i(q_i). \quad (55)$$

Joint constraints, link collision avoidance (Khatib, 1986), and control of manipulator postures can be integrated naturally in the vector Γ_{i_o}.

6 Conclusion

The augmented object model proposed in this paper constitutes a natural framework for the dynamic modelling and control of multi-effector/object systems. In this approach, the control structure only uses the necessary forces, *i.e.* net force, required to achieve the dynamic decoupling and control of the system. Compared to control structures where joint motions or effector motions are individually decoupled and controlled, the proposed control system presents a significant reduction in actuator activities. Indeed, in this approach, the inertial coupling, centrifugal, and Coriolis forces acting on one effector are used to compensate for parts of the coupling forces acting on the others. The actuator joint force activity is further minimized by the criterion used for the allocation of effector forces.

For redundant mechanism systems, the multi-effector/object equations of motion have been established, and a similar control system for dynamic decoupling and control has been developed. The expression of joint forces acting in the nullspace of the Jacobian matrix and consistent with the inertial characteristics perceived by each manipulator has been identified and used for the asymptotic stabilization and gravity compensation of the redundant mechanism.

The methodology developed in this framework constitutes a powerful tool for dealing with the problem of object manipulation in a multi-fingered hand system.

Acknowledement

The financial support of the Systems Development Foundation and the Stanford Institute for Manufacturing and Automation (SIMA) are gratefully acknowledged. The author is grateful to Shashank Shekhar who has made valuable suggestions during the development of this work. Many thanks to Harlyn Baker for careful reading of the manuscript and to Helene Chochon for the drawing.

References

Alford, C. O. and Belyeu, S. M. 1984 (March). Coordinated Control of Two Robot Arms. Proceedings of the IEEE International Conference on Robotics and Automation, Atlanta, Georgia, pp. 468-473.

Hartenberg, R. S. and Denavit, J. 1964. Kinematic Synthesis of Linkages. McGraw-Hill, Inc.

Hayati, S., 1986 (April). Hybrid Position/Force Control of Multi-Arm Cooperating Robots. Proceedings of the IEEE International Conference on Robotics and Automation, San Francisco, California, pp. 1375-1380.

Khatib, O. 1980. Commande Dynamique dans l'Espace Opérationnel des Robots Manipulateurs en Présence d'Obstacles. Thése de Docteur-Ingénieur. École Nationale Supérieure de l'Aéronautique et de l'Espace (ENSAE). Toulouse, France.

Khatib, O. and Burdick, J. 1986 (April). Motion and Force Control of Robot Manipulators. Proceedings of the 1986 IEEE International Conference on Robotics and Automation, pp. 1381-1386, April 7-11, 1986, San Francisco.

Khatib, O. 1986 (Spring). Real-Time Obstacle Avoidance for Manipulators and Mobile Robots. International Journal of Robotic Research, vol. 5, no. 1, pp. 90-98.

Khatib, O. 1987 (February). A Unified Approach to Motion and Force Control of Robot Manipulators: The Operational Space Formulation. IEEE Journal on Robotics and Automation, vol. 3, no. 1.

Tarn, T. J., Bejczy, A. K., and Yun, X., 1987 (March). Design of Dynamic Control of Two Cooperating Robot Arms: Closed Chain Formulation. Proceedings of the IEEE International Conference on Robotics and Automation, Raleigh, North Carolina, pp. 7-13.

Zheng, Y. F. and Luh, J. Y. S., 1986 (April). Joint Torques for Control of Two Coordinated Moving Robots. Proceedings of the IEEE International Conference on Robotics and Automation, San Francisco, California, pp. 1375-1380.