# Isotropic and Uniform Inertial and Acceleration Characteristics: Issues in the Design of Redundant Manipulators

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## Abstract

The paper investigates the dynamic characterization of redundant manipulators and formalizes the problem of dynamic optimization in manipulator design. The dynamic performance of a manipulator is described by both inertial and acceleration characteristics as perceived at the end-effector operational point. The inertial characteristics at this point are given by the operational space kinetic energy matrix (pseudo-kinetic energy matrix for a redundant manipulator) which is dependent on the kinematic and inertial parameters of the manipulator and varies with its configuration. The acceleration characteristics of the end-effector are described by a joint torque/acceleration transmission matrix. In addition to their dependency on the kinematic and inertial parameters, the acceleration characteristics depend on the velocities and actuator torque bounds. The dynamic optimization is formalized in terms of finding the design parameters under the various constraints to achieve the smallest most isotropic and most uniform end-effector inertial properties, while providing the largest, most isotropic, and most uniform bounds on the magnitude of end-effector acceleration. This approach is used in the design of ARTISAN, a ten-degree-of-freedom manipulator currently under development at Stanford University.

## Introduction

Over the past two decades, an important research effort has been devoted to the development of robot systems. This effort has produced significant improvements in dexterity, workspace, and kinematic characteristics of robot mechanisms. Research in kinematics has developed means for the analysis of workspace characteristics [8,9] , and the evaluation of kinematic performance [2,6,11].

Manipulators are highly nonlinear and coupled systems. During motion a manipulator is subject to inertial, centrifugal, and Coriolis forces. The magnitude of these dynamic forces cannot be ignored when large accelerations and fast motions are considered. The dynamic characterization is, therefore, an essential consideration in the analysis, design, and control of these mechanisms. One of the most significant characteristics in evaluating manipulator performance is associated with the dynamic behavior

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of its end-effector. The end-effector is indeed the part most closely linked to the task. These characteristics cannot be found in the manipulator joint space dynamic model, as it provides a description of joint motion dynamics. The description, analysis and control of manipulator systems with respect to the dynamic characteristics of their end-effectors has been the basic motivation in the development of *the operational space formulation* [3,5]. The end-effector dynamic model is a fundamental tool for the analysis and dynamic characterization of manipulator systems.

The inertial characteristics at some point on the end-effector or the manipulated object are given by the operational space kinetic energy matrix. The kinetic energy matrix, or the generalized inertia ellipsoid [1], establishes the relationship between end-effector forces and accelerations. However, this relationship does not relate the actual actuator torque input to the end-effector accelerations. The description of the acceleration characteristics is an essential requirement for the evaluation of the dynamic performance of manipulators. The operational space dynamic model has been used to establish [4], for different regimes, the input/output relationships between joint forces and end-effector acceleration. A similar relationship has been used to establish a measure of dynamic manipulability [12].

The joint torque/acceleration transmission matrix has been used in the design of manipulators with improved dynamic characteristics. An optimal selection of the design parameters has been shown [4] to significantly improve the end-effector dynamic characteristics by providing large, isotropic, and uniform end-effector accelerations.

In this paper, the dynamic characterization integrates both inertial and acceleration properties. The dynamic optimization is aimed at obtaining the smallest, most isotropic and most uniform end-effector inertial characteristics, while providing the largest, most isotropic, and most uniform bounds on the magnitude of end-effector acceleration. The approach is extended to redundant manipulator systems and used in the design of ARTISAN, a ten-degree-of-freedom redundant manipulator.

## End-Effector Equations of Motion

The end-effector position and orientation, with respect to an inertial reference frame $\mathcal{R}_O$ is described by the relationship between $\mathcal{R}_O$ and a coordinate frame $\mathcal{R}_\odot$ of origin $\odot$ attached to this effector. $\odot$ is called the *operational point*. It is with respect to this point that motions and active forces of the effector are specified. An ***operational** coordinate system* associated with an $m$-degree-of-freedom effector and a point $\odot$, **is** a set $\mathbf{x}$ of $m$ *independent* parameters describing the effector position and orientation in $\mathcal{R}_O$. For a non-redundant $n$-degree-of-freedom manipulator, *i.e.* $n = m$, these parameters form a set of of *generalized operational coordinates*. The effector equations of motion in operational space [3,5] are given by

$$\Lambda(\mathbf{x})\ddot{\mathbf{x}} + \mu(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}) = \mathbf{F}; \tag{1}$$

where $\Lambda(\mathbf{x})$ designates the kinetic energy matrix, and $\mathbf{p}(\mathbf{x})$ and $\mathbf{F}$ are respectively the gravity and the generalized operational force vectors. $\mu(\mathbf{x}, \dot{\mathbf{x}})$ represents the vector of centrifugal and Coriolis forces. The dynamic decoupling and motion control of the manipulator in operational space is achieved by selecting the control structure

$$\mathbf{F} = \Lambda(\mathbf{x})\mathbf{F}^* + \mu(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}); \tag{2}$$

and the end-effector becomes equivalent to a *single unit mass*, $I_m$, moving in the $m$-dimensional space,

$$I_m\ddot{\mathbf{x}} = \mathbf{F}^*. \tag{3}$$

$\mathbf{F}^*$ is the input of the decoupled end-effector. This provides a general framework for the implementation of various control structures at the level of decoupled end-effector. The generalized joint forces $\Gamma$ needed to produce the operational forces $\mathbf{F}$ of (eq. 2) are given, using the Jacobian matrix $J(\mathbf{q})$, by

$$\Gamma = J^T(\mathbf{q})\mathbf{F}; \tag{4}$$

where $\mathbf{q}$ represents the vector of generalized joint coordinates.

## Redundant Manipulators

A set of operational coordinates, which describes the end-effector position and orientation, is not sufficient to completely specify the configuration of a redundant manipulator. Therefore, the dynamic behavior of the entire system cannot be described by a dynamic model in operational coordinates. With respect to a system of generalized joint coordinates, the equations of motion of a manipulator can be written in the form

$$A(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \Gamma; \tag{5}$$

where $\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$, and $\Gamma$, represent the Coriolis and centrifugal, gravity, and generalized forces in joint space; and $A(\mathbf{q})$ is the $n \times n$ joint space kinetic energy matrix.

While the dynamics of the entire system cannot be described in operational coordinates, the dynamic behavior of the end-effector itself, can still be described, and its equations of motion in operational space can still be established. In fact, the structure of the effector dynamic model is identical to that obtained in the case of non-redundant manipulators (eq. 1). In the redundant case, however, the matrix $\Lambda$ should be interpreted as a "*pseudo kinetic energy matrix*". This matrix is related to the joint space kinetic energy matrix by $\Lambda = [JA^{-1}J^T]^{-1}$.

Another important characteristic of redundant manipulator is concerned with the relationship between operational forces and joint forces. In the case of non-redundancy, an operational force vector $F$ is produced by the joint force vector $J^T F$. The additional freedom of redundant mechanism results in infinities of possible joint force vectors $\mathbf{\Gamma}$. However, for a given $\mathbf{F}$, all possible joint forces $\mathbf{\Gamma}$ satisfy the relation

$$\mathbf{F} = \bar{J}^T \mathbf{\Gamma}; \tag{6}$$

where

$$\bar{J}(\mathbf{q}) = A^{-1}(\mathbf{q}) J^T(\mathbf{q}) \Lambda(\mathbf{q}). \tag{7}$$

$\bar{J}(\mathbf{q})$ is actually a generalized inverse of the Jacobian matrix. A joint force vector $\mathbf{\Gamma}$ can then be decomposed into two terms: one contributes to the operational force vector, and the other only acts internally (in the null space associated with the Jacobian matrix)

$$\mathbf{\Gamma} = J^T(\mathbf{q})\mathbf{F} + [I_n - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\mathbf{\Gamma}_o; \tag{8}$$

where $I_n$ is the $n \times n$ identity matrix and $\mathbf{\Gamma}_o$ is an arbitrary joint force vector. It has been shown that a generalized inverse that is consistent with the system's dynamics is unique [5] and given by (eq. 7). This generalized inverse corresponds to the solution that minimizes the manipulator's instantaneous kinetic energy.

The relationships between the components of the operational space and joint space dynamic models are

$$\Lambda(\mathbf{q}) = [J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q})]^{-1}; \tag{9}$$

$$\mu(\mathbf{q}, \dot{\mathbf{q}}) = \bar{J}^T(\mathbf{q})\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) - \Lambda(\mathbf{q})\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}); \tag{10}$$

$$\mathbf{p}(\mathbf{q}) = \bar{J}^T(\mathbf{q})\mathbf{g}(\mathbf{q}); \tag{11}$$

where $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{J}(\mathbf{q})\dot{\mathbf{q}}$. The previous relationships are general. In particular, they still apply to non-redundant mechanisms. In this case of zero degree of redundancy, the matrix $\bar{J}$ reduces to $J^{-1}$.

Similar to the case of non-redundant manipulators, the dynamic decoupling and control of the end-effector can be achieved by selecting an operational command vector of the form (eq. 2). The manipulator joint motions produced by this command vector are those that minimize the instantaneous kinetic energy of the mechanism. Asymptotic stabilization is achieved by the addition of dissipative joint forces. In order to preclude any effect of the additional forces on the end-effector and maintain its dynamic decoupling, these forces are selected to act in the dynamically consistent nullspace associated with $\overline{J}(\mathbf{q})$. In the actual implementation, the control vector is developed in a form [5] that avoids the explicit evaluation of the expression of the generalized inverse of the Jacobian matrix.

## End-Effector Dynamic Performance

The dynamic response of a mechanical system is determined by its inertial characteristics. Reducing the magnitude of inertias improves the system's dynamic response. The end-effector inertial characteristics at a configuration $\mathbf{q}$ are described by the kinetic energy matrix $\Lambda(\mathbf{q})$. It's effective inertia at a configuration $\mathbf{q}$, when moving in a direction $\mathbf{u}$ is given by $\mathbf{u}^T\Lambda(\mathbf{q})\mathbf{u}$. The effective inertia varies with the configuration and direction. Isotropic and uniform inertial characteristics are therefore essential to provide isotropic and uniform end-effector's dynamic response.

The second characteristic is concerned with the acceleration characteristics at the end-effector. This is the minimum achievable acceleration given the bounds on actuator torques. Equivalently, this characteristic can be stated in terms of the bounds on the operational force vector $\mathbf{F}^*$, the input of the decoupled end-effector in (eq. 3). Let us examine the operational command vector $\mathbf{F}$ in (eq. 2), which achieves the dynamic decoupling and control of end-effector motion. Only a fraction of these operational forces, namely $\mathbf{F}^*$ the input of the decoupled end-effector, contributes to the end-effector acceleration. The end-effector dynamic performance is, therefore, dependent on the extent of the boundaries of $\mathbf{F}^*$, which determine the limitations on the magnitude of available end-effector acceleration.

The vector $\mathbf{F}$ of (eq. 2) is produced from the actuator joint force vector $\mathbf{\Gamma}$ by $\overline{J}^T(\mathbf{q})\mathbf{\Gamma}$, $\overline{J}(\mathbf{q})$ is equal to $J^{-1}(\mathbf{q})$ for a non-redundant manipulator. Substituting in (eq. 2) yields,

$$\overline{J}^T(\mathbf{q})\mathbf{\Gamma} = \Lambda(\mathbf{q})\mathbf{F}^* + \mu(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{p}(\mathbf{q});$$

which, using (eq. 9- 11), can be written as

$$\mathbf{F}^* = E(\mathbf{q})[\mathbf{\Gamma} - \overline{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}}) - \overline{\mathbf{g}}(\mathbf{q})]; \tag{12}$$

where

$$E(\mathbf{q}) = J(\mathbf{q})A^{-1}(\mathbf{q}). \tag{13}$$

and

$$\overline{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}}) = [J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\ \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) - J^T(\mathbf{q})\Lambda(\mathbf{q})\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}); \tag{14}$$

$$\overline{\mathbf{g}}(\mathbf{q}) = [J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\ \mathbf{g}(\mathbf{q}). \tag{15}$$

$\overline{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}})$ and $\overline{\mathbf{g}}(\mathbf{q})$ are the joint force vectors corresponding to the end-effector Coriolis and centrifugal forces, and Gravity forces. For a non-redundant manipulator, $[J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]$ reduces to the identity matrix and $\overline{\mathbf{g}}(\mathbf{q})$ becomes identical to $\mathbf{g}(\mathbf{q})$. For a redundant manipulator, $\overline{\mathbf{g}}(\mathbf{q})$ reperesents the part of $\mathbf{g}(\mathbf{q})$ that has a contribution at the end-effector, $\overline{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}})$ is similarly interpreted. Given (eq. 3), the matrix $E(\mathbf{q})$ also establishes the relationship between joint torques and accelerations.

$$\ddot{\mathbf{x}} = E(\mathbf{q})\overline{\Gamma}; \tag{16}$$

where

$$\overline{\Gamma} = \Gamma - \overline{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}}) - \overline{\mathbf{g}}(\mathbf{q}). \tag{17}$$

$\overline{\Gamma}$ represents the vector of joint forces that contributes to the end-effector accelerations. These contributing forces are limited by the boundaries of actuator torques. At zero velocity the matrix $E(\mathbf{q})$ describes the bounds on the end-effector accelerations corresponding to the bounds on joint actuator torques corrected for the gravity. The bounds on $\overline{\Gamma}$ has been used [4] to construct a joint force normalization matrix $N_0(\mathbf{q})$. This matrix has been used to define

$$E_0(\mathbf{q}) = WE(\mathbf{q})N_0(\mathbf{q}); \tag{18}$$

where $W$ is a weighting matrix for the normalization of angular and linear accelerations. The matrix $E_0(\mathbf{q})$ can be interpreted as a joint force/acceleration transmission matrix at zero-velocity. Bounds on actuator torques are modified at non-zero velocities. Coriolis and centrifugal forces that arise at non-zero velocities also affect the bounds on $\overline{\Gamma}$. Similarly to $E_0(\mathbf{q})$, a matrix $E_v(\mathbf{q})$

$$E_v(\mathbf{q}) = WE(\mathbf{q})N_v(\mathbf{q}); \tag{19}$$

has been constructed to describe the joint force/acceleration transmission at maximum operating velocities. At a given configuration $\mathbf{q}$, the end-effector's acceleration characteristics will be described by the matrices $E_0(\mathbf{q})$ and $E_v(\mathbf{q})$.

## Dynamic Optimization

The dynamic optimization is aimed at finding the design parameters under the various constraints to achieve the smallest, most isotropic, and most uniform end-effector inertial properties, while providing the largest, most isotropic, and most uniform bounds on the magnitude of end-effector acceleration, or equivalently, on the command vector $\mathbf{F}^*$ both at low and high velocities. The performance at high velocity is important for fast and gross motion, while performance at low velocity is particularly important for fast response in tasks with small range of motion, such as part-mating operations.

At a given configuration $\mathbf{q}$, the matrices $\Lambda(\mathbf{q})$, $E_0(\mathbf{q})$, and $E_v(\mathbf{q})$ are functions of the manipulator's geometric and motion parameters; *e.g.* link length, mass, moment of inertia, centers of mass, actuator mass, and bounds on actuator torques. Let $\eta$ designate the set of these parameters.

The design process would typically start with an initial design based on workspace and geometric considerations. The various design parameters would be estimated within some range. These specifications and the dynamic and structural requirements form the set of design parameters $\eta$. Let $\{u_i(\eta); i = 1, \ldots, n_u\}$ and $\{v_i(\eta); i = 1, \ldots, n_v\}$ designate the sets of equality and inequality constraints on the manipulator design parameters $\eta$.

Expressed as a function of the manipulator configuration $\mathbf{q}$ and the design parameters $\eta$, the matrices $\Lambda(\mathbf{q})$, $E_0(\mathbf{q}, \eta)$ and $E_v(\mathbf{q}, \eta)$ constitute the basic components in this optimization problem. At a given configuration, the problem is to find the optimal design parameters $\eta$, under the constraints $\{u_i(\eta)\}$ and $\{v_i(\eta)\}$, that minimize some cost function based on the end-effector inertial and acceleration characteristics. This cost function is made up of three weighted components associated with the characterisitics of the matrices $\Lambda(\mathbf{q})$, $E_0(\mathbf{q})$, and $E_v(\mathbf{q})$,

$$\mathcal{C}(\mathbf{q}, \eta) = \sum_{i=1}^{3} w_i \mathcal{C}_i(\mathbf{q}, \eta);$$

subject to the equality and inequality constraints

$$u_i(\eta) = 0 \quad i = 1, \ldots, n_u;$$

$$v_i(\eta) \leq 0 \quad i = 1, \ldots, n_v;$$

where $w_i$ are the weight coefficients. The cost function associated with the kinetic energy matrix is aimed at providing small and isotropic inertial properties at $\mathbf{q}$. The magnitude characteristics is described by the norm $\|\Lambda(\mathbf{q})\|$, and the isotropic properties are represented by the matrix condition number, i.e. $\kappa(\Lambda(\mathbf{q}, \eta))$. The first component becomes

$$\mathcal{C}_1(\mathbf{q}, \eta) = [\|\Lambda(\mathbf{q}, \eta)\| + \alpha_1 \kappa(\Lambda(\mathbf{q}, \eta))];$$

The cost functions associated with the end-effector accelerations at zero and maximum oper ting velocity are aimed at providing the largest and most isotropic properties at

q. This is

$$\mathcal{C}_2(\mathbf{q},\eta) = [\frac{1}{\|E_0(\mathbf{q},\eta)\|} + \alpha_2\kappa(E_0(\mathbf{q},\eta))];$$

$$\mathcal{C}_3(\mathbf{q},\eta) = [\frac{1}{\|E_v(\mathbf{q},\eta)\|} + \alpha_3\kappa(E_v(\mathbf{q},\eta))].$$

where $\alpha_1$, $\alpha_2$, $\alpha_3$. Finally, the problem of dynamic optimization over the manipulator work space $\mathcal{D}_\mathbf{q}$ can be expressed as

$$\text{minimize} \int_{\mathcal{D}_\mathbf{q}} \mathcal{C}(\mathbf{q},\eta)w(\mathbf{q})d\mathbf{q};$$

subject

$$u_i(\eta) = 0 \quad i = 1,\ldots,n_u;$$

$$v_i(\eta) \leq 0 \quad i = 1,\ldots,n_v;$$

where the function $w(\mathbf{q})$ is used to relax the weighting of the cost function $\mathcal{C}(\mathbf{q},\eta)$ in the vicinity of the work space boundaries and singularities.

## Application to ARTISAN

Optimal dynamic characteristics at the end-effector has been one of the basic goals in the ARTISAN project [7]. These include high performance joint torque control ability, motion redundancy, micro-manipulation ability [10], light structure, and integrated sensing. The kinematic structure of the ARTISAN is divided into three subsystems: wrist positioning structure, wrist and micro-manipulator. The wrist positioning structure is the part of the manipulator composed of the first four joints. Joint 1 and joint 2 are intersecting, orthogonal revolutes. Joints 3 and 4 are revolutes with axes parallel to the axis of joint 2. This part of the system forms a redundant structure if we regard This part of the system forms a redundant structure with respect to the positioning of the wrist point. The dynamic optimization has been applied to the design of the redundant structure formed by the first four degrees of freedom of ARTISAN.

The design parameters consisted of the links' dimensions, masses, inertias, and motor parameters. The dynamic optimization was conducted in three main steps. Based on the preliminary design, the inertial characteristics were first optimized. This resulted in an initial selection of dimensions and mass distribution. This first set of design parameters is used to initialize, the second step which is aimed at providing optimal acceleration characteristics. Actuators are chosen in this second step. The overall optimization is achieved in the third step.

This procedure, illustrated in Fig. 1., has led to a significant reduction of the search space in steps 1 and 2 and provided a good initial estimate for the overall optimization in step 3. It is important to mention the impact of the various weights on the final solution.

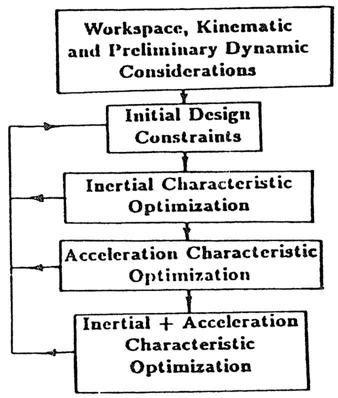


Fig. 1. The Three Step Optimization Procedure

The optimization was carried out using a sequential quadratic programming (SQP) algorithm. The results of this optimization for ARTISAN has been compared to a PUMA 560 arm. Fig. 2. shows the inertial characteristics of the PUMA arm (Fig. 2.a.) and ARTISAN (Fig. 2.b). At a given position of the end-effector, these figures show the projections of the ellipsoids associated with the three eigenvalues of $\Lambda$. Because of the redundancy, different ellipsoids would result at given end-effector poistion. The ellipsoids shown in Fig. 2.b. correspond to those that have the largest eigenvalues. Also, the scale used in Fig. 2.b. is twice that of Fig. 2.a. The average effective inertia of the PUMA is roughly three times that of ARTISAN.

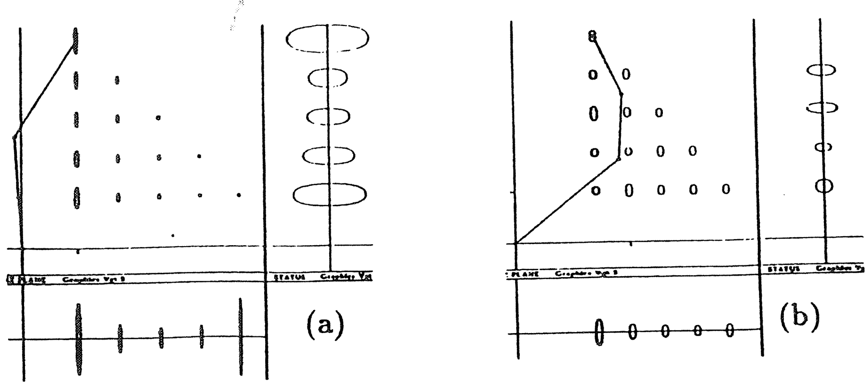


Fig. 2. The inertial Characteristics

Fig. 3. illustrates the minimum available end effector acceleration for the PUMA (Fig. 3.a.) and ARTISAN (Fig. 3.b) at zero joint velocity. The circles depict the min-

imum available accelerations at points in the workspace. On an average, the minimum available accelerations for ARTISAN is twice that of the PUMA arm for same joint torques.

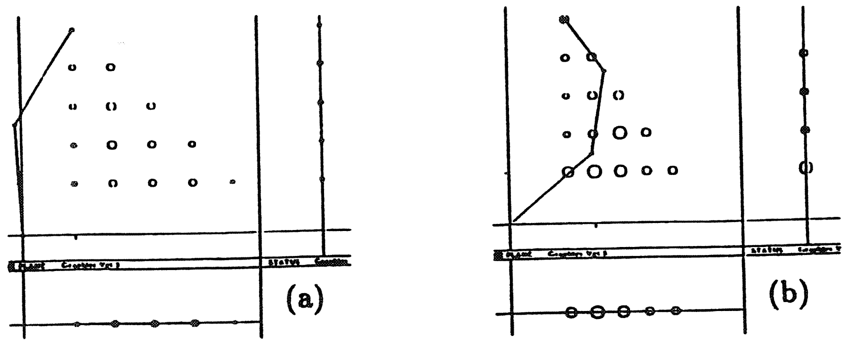


Fig. 3. Minimum Available End-Effector Accelerations

Fig. 4. shows the condition numbers of the acceleration characteristics at zero joint velocity for ARTISAN to be uniform over the workspace. These characteristics has been estimated to be roughly half of those computed for PUMA arm.

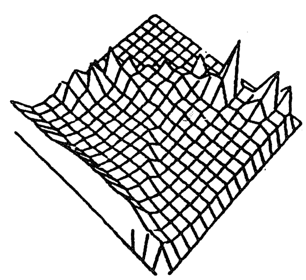

Fig. 4. Acceleration Characteristics

## Conclusion

The dynamic characterisitics of manipulator systems have been described by the inertial and acceleration properties as perceived at their end-effectors. These characterisitics have been used in the developement of a methodology for the dynamic optimization in manipulator design. The optimization problem has been expressed as the minimization, with respect to the design parameters and constraints, of a cost function based on these characteristics. The small isotropic and uniform inertial characteristics will provide higher dynamic response at the end-effector. The large isotropic and uniform bounds on the end-effector accelerations will be translated into a large and well conditioned operational space command vector. The application to ARTISAN has demonstrated the effectiveness of this methodology to provide higher dynamic characteristics. With an optimal redistribution of masses, dimensions, and actuators, the resulting design has been shown to be significantly superior to conventional designs.

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