Reduced Effective Inertia in Macro-/Mini-Manipulator Systems

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The paper investigates the dynamic characteristics of redundant manipulators involving lightweight mechanical structures, e.g. mini-manipulator. The effective inertia of a macro-/mini-manipulator system is shown to be upper bounded by the inertial properties of the lightweight mini-manipulator. In the case of prismatic structures, the inertial properties of the combined system and the lightweight mechanism are shown to be identical. A dextrous dynamic coordination for high bandwidth control of macro-/mini-manipulator mechanisms is proposed. In this approach, the combined system is treated as a single redundant manipulator controlled within the operational space framework. The spatial dexterity is achieved by using the internal motions associated with the redundancy of the mechanism to minimize the deviation from the neutral (mid-range) joint positions of the mini-manipulator. That is to instantaneously maximize the available range of motion of the lightweight structure. The dynamic effects of internal motions on the end-effector are eliminated by the use of joint forces selected from a dynamically consistent null space.

1 Introduction

This paper presents the second fold in our investigation of inertial characteristics of robot mechanisms with combined structures. Our study of systems involving in-parallel combinations of mechanical structures, e.g. multiple manipulators, has shown (Khatib 1988) the inertial characteristics to possess additive properties. The effective inertia expressed at some operational point of the combined system, i.e. object and end-effectors, is given by the sum of the effective inertias associated with the object and the individual mechanical structures, all expressed at the same point. The dynamics of a multi-effector/object system is described by an *augmented object* model, which represents the total mass and inertia perceived at the operational point and actuated by the total effector forces acting at that point.

In this paper, the focus is on the mass and inertial properties of mechanisms resulting from serial combinations of structures, e.g. macro-/mini-manipulator. The study of this type of mechanism falls in the general area of redundant manipulators.

Redundancy is clearly important for extending a manipulator's capabilities to reach within a cluttered workspace (Hanafusa, Yoshikawa, and Nakamura 1981). But beyond enhancing kinematic and workspace characteristics, motion redundancy could have also a significant impact on the manipulator's dynamic performance. The capability of a manipulator to perform fine motions can be significantly improved by incorporating a set of small lightweight links, a mini-manipulator, into the manipulator mechanism (Hollis, 1985; Reboulet and Robert 1986; Tilley, Cannon, and Kraft 1986; Cai and Roth, 1987; Sharon, Hogan, and Hardt, 1988). Clearly, the high accuracy and greater speed of a mini-manipulator is useful for small range motion operations during which the macro-manipulator is held motionless. During force control operations, a mini-manipulator can also be used to overcome manipulator errors in the directions of active force control by using end-effector force sensing to perform small and fast adjustments.

In this paper, we are concerned with the role of dynamic characteristics aspect in the analysis and control of macro-/mini-manipulators. Tasks under consideration extend beyond those relating to small range of motion. This raises important questions regarding the type of workspace needed for optimal use of the fast dynamics of the mini-structure.

2 Inertial Properties

The inertial properties of a manipulator are generally expressed with respect to its motion in joint space. For an n degree-of-freedom manipulator, this is the $n \times n$ configuration dependent matrix, $A(\mathbf{q})$, associated with the quadratic form of its kinetic energy, $1/2\ \dot{\mathbf{q}}^T A(\mathbf{q})\dot{\mathbf{q}}$, where $\mathbf{q}$ and $\dot{\mathbf{q}}$ are the vectors of joint coordinates and joint velocities, respectively.

When concerned with the dynamic response or impact forces at a given point on the end-effector (or manipulated object), the inertial properties involved are those evaluated at that point, termed the *operational point*. Attaching a coordinate frame at the operational point and

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using the relationships between this frame and some reference frame provides a description of the configuration (position and orientation) of the effector.

The number, m, of independent parameters needed to describe the position and orientation of the end-effector determines the number of degrees of freedom the end-effector possesses. When the effector and manipulator have both the same number of degrees of freedom ($n = m$), the independent parameters (*operational coordinates*) form a set of generalized coordinates, $\mathbf{x}$. In this case (that of non-redundant manipulators), the kinetic energy of the mechanism is a quadratic form of (generalized) operational velocities, $1/2\ \dot{\mathbf{x}}^T \Lambda(\mathbf{x})\dot{\mathbf{x}}$, where $\Lambda(\mathbf{x})$ is the $m \times m$ kinetic energy matrix which describes the effector's inertial properties. The identity between the above form and the expression of the kinetic energy in terms of joint velocities and the use of the Jacobian matrix, $J(\mathbf{q})$, which establishes the relationships between joint velocities and effector velocities, yields

$$\Lambda(\mathbf{x}) = J^{-T}(\mathbf{q})A(\mathbf{q})J^{-1}(\mathbf{q}).$$

This matrix, along with its partial derivatives with respect to the operational coordinates (coefficients of centrifugal and Coriolis forces), and the gravity forces acting at the operational point, establishe the equations of motion (Khatib 1980) for the effector subjected to operational forces. Another representation of a manipulator's inertial properties is the generalized inertia ellipsoid (Asada 1983), which uses the inverse of the matrix $\Lambda(\mathbf{x})$. By the nature of coordinates associated with spatial rotations, operational forces acting along rotation coordinates are not homogeneous with respect to moments and vary with the type of representation used. While this characteristic does not raise any difficulty in motion control, the homogeneity issue is important in tasks where both motions and active forces are involved. This issue is also a concern in the analysis of inertial properties. These properties are, in fact, expected to be independent of the type of representation use in the description of the end-effector rotations. The homogeneity issue is addressed by using the relationships between operational velocities and instantaneous angular velocities. The Jacobian matrix $J(\mathbf{q})$ associated with a given selection, $\mathbf{x}$, of operational coordinates can be expressed (Khatib 1987) as

$$J(\mathbf{q}) = E(\mathbf{x})J_O(\mathbf{q});$$

where the matrix $J_O(\mathbf{q})$, termed *the basic Jacobian*, is defined independently of the particular set of parameters used to describe the end-effector configuration, while the matrix $E(\mathbf{x})$ is dependent upon those parameters. The basic Jacobian establishes the relationships between generalized joint velocities $\dot{\mathbf{q}}$ and end-effector linear and angular velocities $\mathbf{v}$ and ω.

Using the basic Jacobian matrix, the mass and inertial properties at the end-effector are described by

$$\Lambda_O(\mathbf{x}) = J_O^{-T}(\mathbf{q})A(\mathbf{q})J_O^{-1}(\mathbf{q}).$$

The effective inertia along a direction in operational space is given by the scalar $\mathbf{u}^T\Lambda_O(\mathbf{x})\mathbf{u}$, where $\mathbf{u}$ is a

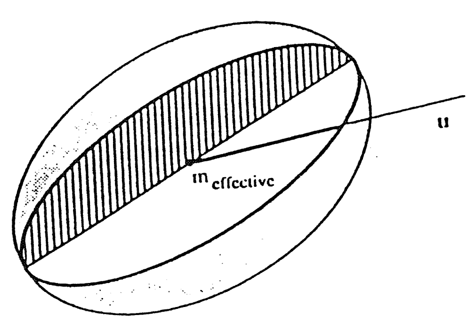

Figure 1: Effective Mass

unit vector describing the direction. In a pure translational motion of the effector, for instance, the quantity $\mathbf{u}^T\Lambda_O(\mathbf{x})\mathbf{u}$, represents the effective mass, $m_{\text{effective}}$, perceived in the direction $\mathbf{u}$, as illustrated in Figure 1.

Redundant Manipulators

A set of operational coordinates, which only describes the end-effector position and orientation, is obviously not sufficient to completely specify the configuration of a redundant manipulator. Therefore, the dynamic behavior of the entire system cannot be described by a dynamic model in operational coordinates. The dynamic behavior of the end-effector itself, nevertheless, can still be described, and its equations of motion in operational space can still be established. In fact, the structure of the effector dynamic model is identical to that obtained in the case of non-redundant manipulators. In the redundant case, however, the matrix Λ_O should be interpreted as a "*pseudo kinetic energy matrix*". This matrix is related to the joint space kinetic energy matrix by

$$\Lambda_O(\mathbf{q}) = [J_O(\mathbf{q})A^{-1}(\mathbf{q})J_O^T(\mathbf{q})]^{-1}.$$

3 macro-/Mini-Manipulator Systems

We now consider the case of systems resulting from serial combinations of two manipulators. The manipulator connected to the ground will be referred to as the "macro-manipulator". It has n_M degrees of freedom and its configuration is described by the system of n_M generalized joint coordinates $\mathbf{q}_M$. The second manipulator, referred to as the "mini-manipulator", has n_m degrees of freedom and its configuration is described by the generalized coordinates $\mathbf{q}_m$. The resulting structure is an n degree-of-freedom manipulator with $n = n_M + n_m$. Its configuration is described by the system of generalized joint coordinates $\mathbf{q} = [\mathbf{q}_M^T\ \mathbf{q}_m^T]^T$. If m represents the number of effector degrees of freedom of the combined structure, n_M and n_m are assumed to obey

$$n_M \geq 1 \text{ and } n_m = m.$$

This assumption states that the mini-manipulator must possess full freedom of motion in the operational space. The macro-manipulator must have at least one degree-of-freedom.

3.1 Kinematics

The kinematics of the two manipulators, considered separately, are described with respect to the reference frames $\mathcal{R}_{M(O)}$ and $\mathcal{R}_{m(O)}$. The coordinate frames associated with their operational points are denoted $\mathcal{R}_M$ and $\mathcal{R}_m$ respectively. The transformation matrix describing the rotation between the frames $\mathcal{R}_{M(O)}$ and $\mathcal{R}_M$ is $S_M(\mathbf{q}_M)$. The operational coordinates are $\mathbf{x}_M$ and $\mathbf{x}_m$, and $J_M(\mathbf{q}_M)$ and $J_m(\mathbf{q}_m)$ are the respective Jacobian matrices. Let $J_{M(O)}(\mathbf{q}_M)$ and $J_{m(O)}(\mathbf{q}_m)$ be the basic Jacobian matrices associated with two individual manipulators; the basic Jacobian matrix associated with the serial combination can be expressed as

$$J_O = [V J_{M(O)} \quad \Omega J_{m(O)}]; \tag{1}$$

where

$$V = \begin{bmatrix} I & -\hat{v} \\ 0 & I \end{bmatrix}; \quad \text{and} \quad \Omega = \begin{bmatrix} S_M & 0 \\ 0 & S_M \end{bmatrix}. \tag{2}$$

$\hat{v}$ is the cross product operator on the position vector associated with the mini-manipulator and expressed in $\mathcal{R}_{M(O)}$. I is the 3×3 identity matrix.

3.2 Dynamics

The kinetic energy matrix, $A(\mathbf{q})$, of the combined system can be decomposed into diagonal blocks corresponding to the dimensions of the two manipulators' individual kinetic energy matrices

$$A(\mathbf{q}) = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}. \tag{3}$$

It can be easily shown that the matrix A_{22} of dimensions $n_m \times n_m$ in equation 3 is identical to the kinetic energy matrix A_m associated with the mini-manipulator considered alone, i.e. $A_{22} = A_m$. The inverse of the kinetic energy matrix $A(\mathbf{q})$ is

$$A^{-1}(\mathbf{q}) = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{12}^T & \overline{A}_{22} \end{bmatrix}. \tag{4}$$

The operational space *pseudo kinetic energy matrix* Λ_O associated with the linear and angular velocities is defined by $(J_O A^{-1} J_O^T)^{-1}$. Using equation 1, the inverse of this matrix can be written as

$$\Lambda_O^{-1} = \Omega \Lambda_{m(O)}^{-1} \Omega^T + \overline{\Lambda}_C; \tag{5}$$

where

$$\overline{\Lambda}_C = J_O \overline{A}_C J_O^T; \tag{6}$$

and

$$\overline{A}_C = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{12}^T & \overline{A}_{22} - A_m^{-1} \end{bmatrix}. \tag{7}$$

<u>Theorem 1</u>: *(Reduced Effective Inertia)* For $k = 1, 2, \ldots, m$, the operational space *pseudo kinetic energy matrices* Λ_O (combined mechanism), and $\Lambda_{m(O)}$ (mini-manipulator) satisfy

$$\frac{1}{1 + \|\overline{\Lambda}_C\| \cdot \lambda_k(\Lambda_{m(O)})} \leq \frac{\lambda_k(\Lambda_O)}{\lambda_k(\Lambda_{m(O)})} \leq 1;$$

where $\lambda_k(.)$ denote the k^{th} largest eigenvalue of (.), i.e. $\lambda_n(.) \leq \ldots \leq \lambda_1(.)$.
This theorem states that the effective inertia, in any direction, of the combined mechanism is smaller than or equal to the inertias associated with the mini-manipulator.
The proof involves the two steps:

<u>Step 1</u>: *(Eigenvalue Characteristic)* This first step is based on an important characteristic of symmetric matrices. It is possible to show that (Wilkinson, 1965): *If M and M+E are n x n symmetric matrices, then for* $k = 1, 2, \ldots, n$

$$\lambda_k(M) + \lambda_n(E) \leq \lambda_k(M + E) \leq \lambda_k(M) + \lambda_1(E);$$

Applying this relation to equation 5 for $k = 1, 2, \ldots, m$, and noting that $\Lambda_{m(O)}$ and $\Omega \Lambda_{m(O)} \Omega^T$ are similar positive definite matrices with the identical eigenvalues $1/\lambda_k(\Lambda_{m(O)}^{-1})$, yields

$$\frac{1}{1+\alpha} \leq \frac{\lambda_k(\Lambda_O)}{\lambda_k(\Lambda_{m(O)})} \leq \frac{1}{1+\beta}. \tag{8}$$

where

$$\begin{aligned} \alpha &= \lambda_1(\overline{\Lambda}_C) \cdot \lambda_k(\Lambda_O L); \\ \beta &= \lambda_n(\overline{\Lambda}_C) \cdot \lambda_k(\Lambda_O L); \end{aligned}$$

<u>Step 2</u>: *(Positive Semidefinition of* $\overline{\Lambda}_C$*)*. Let us consider the symmetric matrix

$$M = \begin{bmatrix} E & C^T \\ C & D \end{bmatrix}.$$

If E is a nonsingular matrix, M can be decomposed (Bruch and Parlett, 1971) following

$$M = \begin{bmatrix} I & 0 \\ CE^{-1} & I \end{bmatrix} \begin{bmatrix} E & 0 \\ 0 & D - CE^{-1}C^T \end{bmatrix} \begin{bmatrix} I & E^{-1}C^T \\ 0 & I \end{bmatrix}. \tag{9}$$

Applying this decomposition to $\overline{A}_C$ ($\overline{A}_{11}$ is nonsingular, since A^{-1} is nonsingular), and using the relationships between the block matrices resulting from $AA^{-1} = I$, yields

$$\overline{A}_C = \begin{bmatrix} I & 0 \\ \overline{A}_{12}^T \overline{A}_{11}^{-1} & I \end{bmatrix} \begin{bmatrix} \overline{A}_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I & \overline{A}_{11}^{-1} \overline{A}_{12}^T \\ 0 & I \end{bmatrix}. \tag{10}$$

Like $\overline{A}$, the diagonal block $\overline{A}_{11}$ is positive definite. The decomposition (10) shows $\overline{A}_C$ to be positive semidefinite,

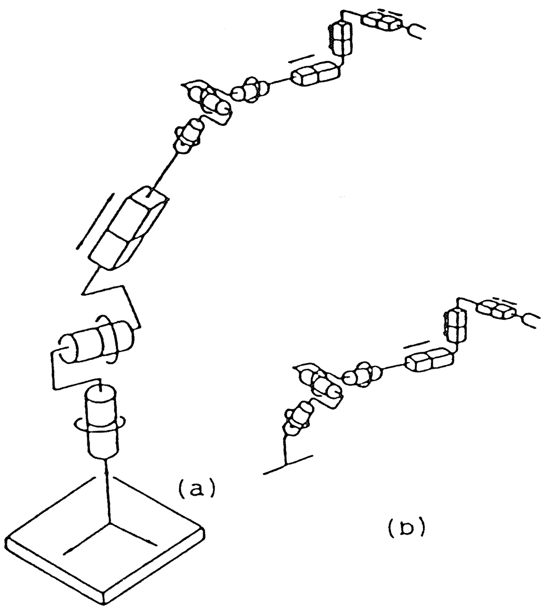

Figure 2: A Macro-/Mini-Structure

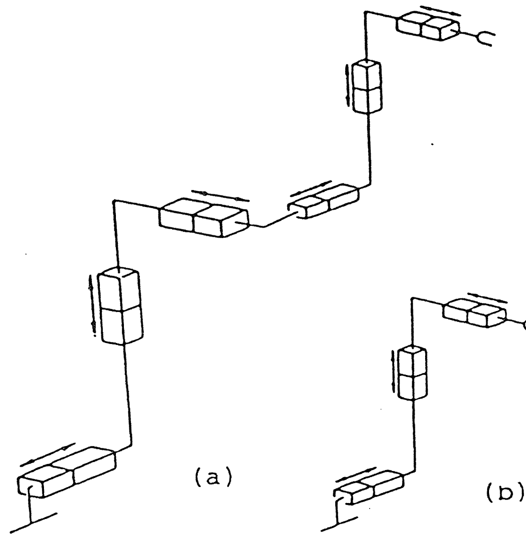

Figure 3: A Prismatic Macro-/Mini-Structure

$\lambda_n(\overline{A}_C) = 0$ and $\lambda_1(\overline{A}_C) \geq 0$. The matrix $\overline{\Lambda}_C$ which results from a congruence transformation (equation 6) of $\overline{A}_C$ is similarly defined, i.e. positive semidefinite with $\lambda_n(\overline{\Lambda}_C) = 0$ and $\lambda_1(\overline{\Lambda}_C) \geq 0$. Substituting this result in equation 8 completes the proof of the Theorem.

Figure 2.a shows a nine degree-of-freedom manipulator. The magnitude of the inertial characteristics of this manipulator are bounded by the inertial characteristics of the six degree-of-freedom mini manipulator shown in Figure 2.b.

Prismatic Manipulators

Let us consider the case of a six-prismatic joint manipulator such as the one shown in Figure 3.a. The operational space associated with this structure is of dimensions 3. The 3×6 Jacobian matrix associated with this manipulator is

$$J(q) = [\, I \quad I \,];$$

where I is the identity matrix.

The kinetic energy matrix associated with this manipulator has the general form

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}.$$

It can be easily shown that $A_{11} = diag(m_i)$ and $A_{22} = diag(m_{i+3})$, where m_i is the mass of the i^{th} link, and $A_{12} = A_{22}$. The inverse of A is

$$A^{-1} = \begin{bmatrix} (A_{11} - A_{22})^{-1} & -(A_{11} - A_{22})^{-1} \\ -(A_{11} - A_{22})^{-1} & (A_{11} - A_{22})^{-1} + A_{22}^{-1} \end{bmatrix}.$$

Using equations 6 and 7, the matrix $\overline{\Lambda}_C$ associated with this manipulator is

$$\overline{\Lambda}_C = [\, I \quad I \,] \begin{bmatrix} (A_{11} - A_{22})^{-1} & -(A_{11} - A_{22})^{-1} \\ -(A_{11} - A_{22})^{-1} & (A_{11} - A_{22})^{-1} \end{bmatrix} \begin{bmatrix} I \\ I \end{bmatrix};$$

which is zero. This leads to

Corollary 1: *(Reduced Inertia/Prismatic Structure)* For a prismatic structure, the operational space *pseudo kinetic energy matrices* Λ_O (combined mechanism), and $\Lambda_{m(O)}$ (mini-manipulator) satisfy

$$\lambda_k(\Lambda_O) = \lambda_k(\Lambda_{m(O)}); \quad k = 1, 2, \ldots, m.$$

Using equation 5 and noting that $\Omega = I$, yields

Corollary 2: *(Pseudo Kinetic Energy Matrix/-Prismatic Structure)* For a prismatic structure, the operational space *pseudo kinetic energy matrices* Λ (combined mechanism), and Λ_m (mini-manipulator) are identical:

$$\Lambda_O = \Lambda_{m(O)}.$$

The inertial characteristics of the manipulator shown in Figure 3.a are identical to those of the three degree-of-freedom mini manipulator shown in Figure 3.b.

4 Dextrous Dynamic Coordination

The previous results show that the inertial characteristics of the combined system are upper bounded by (and, for prismatic structure, identical to) those of the mini-manipulator. Given the mechanical limits on the range of joint motions of the mini-manipulator, these characteristics are only useful within this available range.

The control in operational space of the macro-/mini-manipulator treated as a single redundant manipulator will result in a fast dynamic response, which will be essentially carried out by the mini-structure. These dynamic characteristics are well maintained until the mini-structure's joints reach their limits. Maximizing the mini-manipulator's available range of motion is therefore essential for extending this performance to tasks with large range of motion.

The proposed *dextrous dynamic coordination* is based on the minimization of the deviation from the neutral (mid-range) joint positions of the mini-manipulator. This minimization is achieved by controlling the manipulator's internal motions, while the end-effector is performing its task. Eliminating the dynamic interaction between these two tasks is a primarily concern.

4.1 Consistent Null Space

End-effector motions are controlled by operational forces, $\mathbf{F}$, created by the application of a set of generalized joint forces, Γ, given by $\Gamma = J^T(\mathbf{q})\mathbf{F}$. For redundant manipulators, the previous relationship becomes incomplete. At a given configuration, there is an infinity of elementary displacements of the redundant mechanism that could take place without altering the configuration of the effector. Those displacements correspond to motion in the null space associated with a generalized inverse of the Jacobian matrix.

In terms of forces, there is also an infinity of joint force vectors that could be applied without effecting the resulting forces reflected at the end-effector. Those are the joint forces acting within the null space. The general expression for the relationship between end-effector forces and generalized joint forces is

$$\Gamma = J^T(\mathbf{q})\mathbf{F} + [I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\Gamma_0; \qquad (11)$$

where Γ_0 is an arbitrary generalized joint force vector. While $\mathbf{F}$ is used for end-effector control, the joint torque vector Γ_0 provides means to control the manipulator internal joint motions.

The previous relationship (11), which is based only on static considerations, provides a freedom in the selection of the generalized inverse ($J^\#$ such that $J = JJ^\# J$). Taking into account the effector's dynamics results in an additional constraint, reducing this freedom. The additional constraint is concerned with end-effector accelerations. Analysis of equations of motion shows that the effector acceleration corresponding to the application of a joint torque vector Γ is $J(\mathbf{q})A^{-1}(\mathbf{q})\Gamma$. In order for the dynamic effects of the joint forces associated with null space to be canceled, it is necessary for the null space to satisfy

$$J^T(\mathbf{q})A^{-1}(\mathbf{q})[I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})]\Gamma_0 = 0. \qquad (12)$$

The null space associated with a generalized inverse satisfing the above constraint is said to be dynamically consistent.

<u>Theorem 3</u>: *(Dynamic Consistency)* A generalized inverse that is consistent with the dynamic constraint of equation 12, $\overline{J}(\mathbf{q})$, is unique and given by

$$\overline{J}(\mathbf{q}) = A^{-1}(\mathbf{q})J^T(\mathbf{q})\Lambda(\mathbf{q}). \qquad (13)$$

$\overline{J}(\mathbf{q})$ in equation 13 is actually a generalized inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's instantaneous kinetic energy. A Joint force vector of the form $[I - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\Gamma_o$ not only corresponds to a zero-vector of operational forces at static equilibrium, but also during motion.

4.2 Internal Motion Control

The joint force vector Γ_o can now be used to maximize the range of motion available at the mini-structure. Let $\overline{q}_i$ and $\underline{q}_i$ be the upper and lower bounds on the i^{th} joint position q_i. We construct the potential function

$$V_{\text{Dextrous}}(\mathbf{q}) = k_d \sum_{i=n_M+1}^{n} (q_i - \frac{\overline{q}_i + \underline{q}_i}{2})^2; \qquad (14)$$

where k_d is a constant coefficient. The gradient of this function

$$\Gamma_{\text{Dextrous}} = -\nabla V_{\text{Dextrous}}; \qquad (15)$$

provides the required attraction (Khatib 1986) to the mid-range joint positions of the mini-manipulator. The interference of these additional torques with the end-effector dynamics is avoided by selecting them from the null space. This is

$$\Gamma_{nd} = [I_n - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\Gamma_{\text{Dextrous}}. \qquad (16)$$

The avoidance of joint limits can be achieved using an "artificial potential field" function. Asymptotic stabilization of the redundant mechanism requires additional dissipative joint forces which should also be selected from the dynamically consistent null space.

It is essential that the range of motion of the joints associated with the mini-structure allows accommodation for the relatively slower dynamic response of the arm. A sufficient motion margin is required for achieving dextrous dynamic coordination.

5 Conclusion

The dynamic analysis of mechanisms with serial combinations of structures has shown the inertial properties to be upper bounded by the properties associated with the set of last links that spans the effector's operational space. The effective inertias of a macro-/mini-manipulator are bounded by those of the lightweight mini-manipulator considered alone.

Treating the manipulator and mini-manipulator as a single redundant system, a *dextrous dynamic coordination* based on minimizing the deviation from the neutral (mid-range) joint positions of the mini-manipulator has been proposed. In order to preclude any effect of the forces used to achieve the spatial dexterity on the primary task, this minimization uses joint forces selected from a dynamically consistent null space. We intend to implement this approach on ARTISAN, a ten degree-of-freedom macro-/mini-manipulator (Roth et al. 1988, Waldron et al. 1987) currently under construction at Stanford.

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