REVIEW

# The Operational Space Framework*

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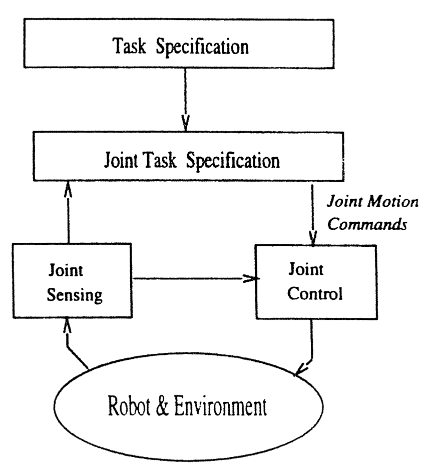
**Robot dynamics has been traditionally viewed from the perspective of a manipulator's joint motions, and significant effort has been devoted to the development of joint space dynamic models and control methodologies. However, the limitations of joint space control techniques, especially in constrained motion tasks, have motivated alternative approaches for dealing with task-level dynamics and control. The operational space formulation, which falls within this line of research, has been driven by the need to develop mathematical models for the description, analysis, and control of robot dynamics with respect to task behavior. In this article, we review the operational space task-level models and discuss the various control methodologies that have been developed in this framework. These include: the unified motion and force control approach; the notion of dynamic consistency in redundant manipulator control; the reduced effective inertia property associated with macro-/mini-manipulator systems and the dextrous dynamic coordination strategy proposed for their control; and the augmented-object model for the control of robot systems involving multiple manipulators.**

*Key Words:* Force Control, Redundancy, Singularities, Multi-Arm Robot System, Macro-/Mini-Manipulator System

## 1. Introduction

The difficulty with joint space control techniques lies in the discrepancy between the space where robot tasks are specified and the space in which the control is taking place. By its very nature, joint space control calls for transformations whereby joint space descriptions are obtained from the robot task specifications. Typically, a joint space control system is organized following the general structure shown in Fig. 1. At the highest level, tasks are specified in terms of end-effector or manipulated object's motion, compliances, and contact forces and moments. Tasks are then transformed at the intermediate level into descriptions in terms of joint positions, velocities, accelerations, compliances, and joint torques. This provides the needed input to the control level, which acts at the robot joints. The task transformation problem associated with joint space control has been the basic motivation for much of the early work in task-level control schemes[(1)-(4)].

**Motion/Force Control**: Beyond the costly transformations it requires, joint space control is incompatible

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with the requirements of constrained tasks, which involve simultaneous motion and force control. Joint space dynamic models provide a description of the dynamic interaction between joint axes. However, what is needed in the analysis and design of force control algorithms is a description of the dynamic interaction between the end-effector or manipulated object and mating parts.

In the absence of such descriptions, most of the research in force control has been driven by kinematic and static considerations. Compliant motion control has been achieved through the use of inner loops of position or velocity control[5],[6]. While position-based or velocity-based compliant motion control have been successfully used in many quasi-static operations, their performance in dynamic tasks has been very limited. The gain limitation associated with these techniques has generally resulted in slow and sluggish behavior. Hybrid position/force control[7] and non-dynamic implementations of impedance control have also resulted in limited dynamic performance.

Dealing with dynamics is essential for achieving higher performance. In free motion, the effects of dynamics increase with the range of motion, speed, and acceleration at which a robot is operating. In part mating operations, dynamics effects also increase with the rigidity of the mating object. Furthermore, control of the end-effector contact forces in some direction is affected by the inertial forces caused by end-effector motion in the subspace orthogonal to that direction. These effects must be taken into account to achieve higher performance.

There is a clear need for the development of dynamic models for robot behavior at the end-effector, manipulated object, or task level. This has been precisely the motivation behind the development of the operational space formulation[8]. In this framework, both motions and active forces are addressed at the same level of end-effector control. The result is a unified approach for dynamic control of end-effector motions and contact forces. This approach is presented in Sec. 4.

**Redundancy and Singularities**: The joint space task transformation problem is exacerbated for mechanisms with redundancy or at kinematic singularities. The typical approach involves the use of pseudo- or generalized inverses to solve an under-constrained or degenerate system of linear equations, while optimizing some given criterion[9]-[11]. Other inverses with improved performance also have been investigated, e.g., the singularity robust inverse[12].

In Sec. 5, we present the extension of the operational space formulation to redundant mechanisms and discuss the control strategy developed in this framework for dealing with kinematic singularities. The control of redundant manipulators relies on two basic models: a task-level dynamic model obtained by projecting the manipulator dynamics into the operational space, and dynamically consistent force/torque relationship that provides decoupled control of joint motions in the null space associated with the redundant mechanism. These two models are also critical for implementing the coordination strategy for macro-/mini-manipulator systems and the control strategy for kinematic singularities. In fact, at singular configurations, a manipulator is treated as a redundant mechanism in the subspace orthogonal to the singular direction.

**Macro-/Mini-Manipulator Systems**: High-performance control of forces and motions requires a robot structure to have a high mechanical bandwidth. Incorporating lightweight links, i.e., a mini-manipulator, at the end of the arm can greatly improve this bandwidth and significantly increase the ability of the manipulator to perform fine motions[13],[14]. Clearly, the higher accuracy and greater speed of a mini-manipulator are useful for small motion operations, during which the rest of the manipulator can be held motionless. In force control operations, a mini-manipulator can be used to overcome manipulator errors in the directions of force control by using end-effector force sensing to perform small and fast adjustments. However, the improvement in dynamic performance with lightweight links is not limited to tasks with small range of motion or to force control operations.

The difficulty with operations covering a wide range of motion is due to the mechanical limits on joint motions of the mini-manipulator. Indeed, high mechanical bandwidth is only available within the range of mini-manipulator motions. An effective strategy for dextrous coordination of motion between the macro and mini structures is therefore essential for maintaining the high bandwidth characteristic of the overall system[15]. In Sec. 6, we discuss the inertial properties of macro-/mini-manipulators and present a general methodology for their coordination and control.

**Multi-Arm Systems**: Another area of growing interest is multi-arm robot systems. Multi-arm control has been generally treated as a motion coordination problem. One of the first schemes for the control of a two-arm system[16] was organized in a master/slave fashion, and used a motion coordination procedure to minimize errors in the relative position of the two manipulators. In another study[17], one manipulator was treated as a "leader" and the other as a "follower". Control of the follower was based on the constraint relationships between the two manipulators. In contrast, the two manipulators were given a symmetric role in the coordination proposed by Uchiyama and Dauchez[18]. The problem of controlling both motion and force in multi-arm systems has been investigated by Hayati[19]. In the proposed approach, the load is partitioned among the arms. Dynamic decoupling and motion control are then achieved at the level of individual manipulator effectors. In the force control subspace, the magnitude of forces is minimized. Tarn, Bejczy, and Yun[20] developed a closed chain dynamic model for a two-manipulator system with respect to a selected set of generalized joint coordinates. Nonlinear feedback and output decoupling techniques were then used to linearize and control the system in task coordinates. In Sec. 7, we present the augmented object model, which extends the operational space approach to multi-arm robot systems.

## 2. Operational Space: Basic Concepts

The basic idea in the operational space approach[8],[21] is to control motions and contact forces through the use of control forces that act directly at the level of the end-effector. These control forces are produced by the application of corresponding torques and forces at the manipulator joints.

For instance, subjecting the end-effector to the gradient of an attractive potential field will result in joint motions that position the effector at the configuration corresponding to the minimum of this potential field. This type of control can be shown to be stable. However, the dynamic performance of such a control scheme will clearly be limited, given the inertial interactions between the moving links.

High performance control of end-effector motions and contact forces requires the construction of a model describing the dynamic behavior as perceived at the end-effector, or more precisely at the point on the effector where the task is specified. This point is called the operational point.

A coordinate system associated with the operational point is used to define a set of operational coordinates. A set of operational forces acting on the end-effector is associated with the system of operational coordinates selected to describe the position and orientation of the end-effector. The construction of the end-effector dynamic model is achieved by expressing the relationship between its positions, velocities, and accelerations, and the operational forces acting on it.

The operational forces are produced by submitting the manipulator to the corresponding joint forces, using a simple force transformation. The use of the forces generated at the end-effector to control motions leads to a natural integration of active force control. In this framework, simultaneous control of motions and forces is achieved by a unified command vector for controlling both the motions and forces at the operational point.

The operational space robot control system is organized in a hierarchical structure, as shown in Fig. 2, of three control levels:

- Task Specification Level: At this level, tasks are described in terms of motion and conatct forces of the manipulated object or tool.
- Effector Level: This level is associated with the end-effector dynamic model, the basis for the control of the end-effector motion and contact forces. The output here is the vector of joint forces and torques to be produced by the joint level in order to generate the operational forces and moments associated with the end-effector control vector.
- Joint Level: This level is formed by the set of individual joint torque controllers, allowing each joint to produce its assigned torque component for producing the vector of joint torques corresponding to the end-effector control vector.

## 3. Operational Space Dynamics

The dynamic equations of a manipulator are generally expressed with respect to its motion in joint space. For an $n$ degree-of-freedom manipulator, the joint space inertial properties are described by the $n\times n$ configuration dependent matrix, $A(\boldsymbol{q})$, associated with the quadratic form of its kinetic energy, $1/2\dot{\boldsymbol{q}}^T A(\boldsymbol{q})\dot{\boldsymbol{q}}$, where $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ are the vectors of joint positions and joint velocities, respectively. The joint space equations of motion may be written

$$A(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{b}(\boldsymbol{q},\dot{\boldsymbol{q}})+\boldsymbol{g}(\boldsymbol{q})=\boldsymbol{\Gamma}; \quad (1)$$

where $\boldsymbol{b}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the vector of centrifugal and Coriolis

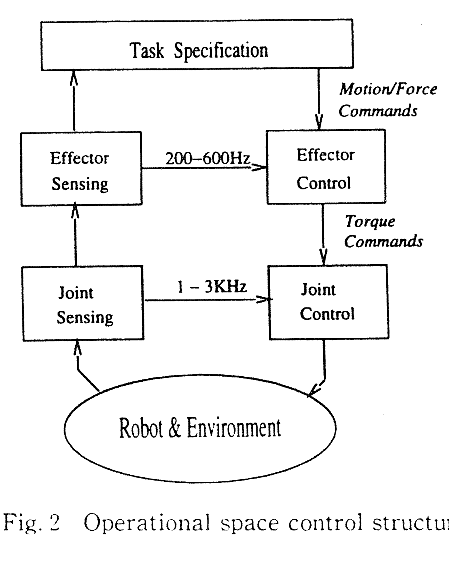


Fig. 2 Operational space control structure

joint-forces and $g(q)$ is the gravity joint-force vector. $\Gamma$ is the vector of generalized joint-forces.

### 3.1 End-effector equations of motion

When the dynamic response or impact force at some point on the end-effector or manipulated object are of interest, the inertial properties involved are those evaluated at that point, termed the operational point. Attaching a coordinate frame to the end-effector at the operational point and using the relationships between this frame and the reference frame attached to the manipulator base provides a description, $x$, of the configuration, i.e. position and orientation, of the effector.

The number, $m$, of independent parameters needed to describe the position and orientation of the end-effector determines its number of degrees of freedom. When the effector and manipulator have both the same degree of freedom, i.e., $n=m$, the operational coordinates, $x$, form a set of generalized coordinates for the mechanism[21] in a domain of the workspace that excludes the kinematic singularities. In this case, the kinetic energy of the mechanism is a quadratic form of the generalized operational velocities, $1/2\dot{x}^T\Lambda(x)\dot{x}$, where $\Lambda(x)$ is the $m\times m$ kinetic energy matrix associated with the operational space.

The operational space kinetic energy matrix $\Lambda(x)$ provides a description of the inertial properties of the manipulator at the operational point. The relationship between the matrices $\Lambda(x)$ and $A(q)$ can be established by stating the identity between the two quadratic forms of kinetic energy and by using the relationship between joint velocities and effector velocities, which involves the Jacobian matrix, $J(q)$. This yields

$$\Lambda(x)=J^{-T}(q)A(q)J^{-1}(q). \tag{2}$$

The matrix $\Lambda(x)$, along with its partial derivatives with respect to the operational coordinates (coefficients of centrifugal and Coriolis forces), and the gravity forces acting at the operational point, establish the equations of motion[8],[21] for the effector subjected to operational forces, $F$. These equations are

$$\Lambda(x)\ddot{x}+\mu(x, \dot{x})+p(x)=F\;; \tag{3}$$

where $\mu(x, \dot{x})$ and $p(x)$ are respectively the centrifugal and Coriolis force vector and the gravity force vector acting in operational space.

### 3.2 Basic dynamic model

By the nature of coordinates associated with spatial rotations, operational forces acting along rotation coordinates are not homogeneous to moments and vary with the type of representation being used (e.g. Euler angles, direction cosines, Euler parameters). While this characteristic does not raise any difficulty in free motion operations, the homogeneity issue is important in tasks where both motions and active forces are involved. This issue is also a concern in the analysis of inertial properties. These properties are, in fact, expected to be independent of the type of representation used for the description of the end-effector orientation.

The homogeneity issue is addressed by using the relationships between operational velocities and instantaneous angular velocities. The Jacobian matrix $J(q)$ associated with a given selection, $x$, of operational coordinates can be expressed[21] as

$$J(q)=E(x)J_0(q)\;; \tag{4}$$

where the matrix $J_0(q)$, termed the basic Jacobian, is defined independently of the particular set of parameters used to describe the end-effector configuration, while the matrix $E(x)$ is dependent upon those parameters. The basic Jacobian establishes the relationships between generalized joint velocities $\dot{q}$ and end-effector linear and angular velocities $v$ and $\omega$.

$$\vartheta\triangleq\begin{bmatrix} v \\ \omega \end{bmatrix}=J_0(q)\dot{q}. \tag{5}$$

Using the basic Jacobian matrix, the mass and inertial properties at the end-effector are described by

$$\Lambda_0(x)=J_0^{-T}(q)A(q)J_0^{-1}(q). \tag{6}$$

The above matrix is related to the kinetic energy matrix associated with a set of operational coordinates, $x$, by

$$\Lambda(x)=E^{-T}(x)\Lambda_0(x)E^{-1}(x). \tag{7}$$

Like angular velocities, moments are defined as instantaneous quantities. A generalized operational force vector $F$ associated with a set of operational coordinates, $x$, is related to forces and moments by

$$F_0\triangleq\begin{bmatrix} F \\ M \end{bmatrix}=E^T(x)F\;; \tag{8}$$

where $F$ and $M$ are the vectors of forces and moments. With respect to linear and angular velocities, the end-effector equations of motion can be written as

$$\Lambda_0(x)\dot{\vartheta}+\mu_0(x, \vartheta)+p_0(x)=F_0\;; \tag{9}$$

where $\Lambda_0(x)$, $\mu_0(x, \vartheta)$, and $p_0(x)$ are defined similarly to $\Lambda(x)$, $\mu(x, \dot{x})$, and $p(x)$ using $J_0(q)$ instead of $J(q)$. In Eq. (9), the dynamics of the end-effector is described with respect to linear and angular velocities. Therefore, a task transformation of the description of end-effector orientation is needed. Such a transformation involves the inverse of $E(x)$ and its derivaties[8].

## 4. Unified Motion and Force Control

Equation (9) is the basis for the development of the unified approach for motion and force control. Compliant motion and part mating operations involve motion control in some directions and force control in orthogonal directions, as illustrated in Fig. 3. Such tasks are described by the generalized selection matrix[21] $\Omega$ and its complement $\bar{\Omega}$ associated with motion control and force control, respectively. Using Eq. ( 9 ), the end-effector/sensor equations of motion can be written as

$$\Lambda_0(\boldsymbol{x})\dot{\vartheta}+\mu_0(\boldsymbol{x},\vartheta)+\boldsymbol{p}_0(\boldsymbol{x})+\bar{\Omega}\boldsymbol{F}_s=\boldsymbol{F}_0. \qquad (10)$$

The vector $\bar{\Omega}\boldsymbol{F}_s$ represents the constraint forces act ing at the end-effector. The unified approach for end-effector dynamic decoupling, motion and active force control is achieved by selecting the control structure

$$\boldsymbol{F}_0=\boldsymbol{F}_{\text{motion}}+\boldsymbol{F}_{\text{active-force}}; \qquad (11)$$

where

$$\boldsymbol{F}_{\text{motion}}=\hat{\Lambda}_0(\boldsymbol{x})\Omega\boldsymbol{F}^*_{\text{motion}}+\hat{\mu}_0(\boldsymbol{x},\vartheta)+\hat{\boldsymbol{p}}_0(\boldsymbol{x}); \qquad (12)$$

$$\boldsymbol{F}_{\text{active-force}}=\hat{\Lambda}_0(\boldsymbol{x})\bar{\Omega}\boldsymbol{F}^*_{\text{active-force}}+\bar{\Omega}\boldsymbol{F}_{\text{desired}}; \qquad (13)$$

where, $\hat{\Lambda}_0(\boldsymbol{x})$, $\hat{\mu}_0(\boldsymbol{x},\dot{\boldsymbol{x}})$, and $\hat{\boldsymbol{p}}_0(\boldsymbol{x})$ represent the estimates of $\Lambda_0(\boldsymbol{x})$, $\mu_0(\boldsymbol{x},\dot{\boldsymbol{x}})$, and $\boldsymbol{p}_0(\boldsymbol{x})$. The vectors $\boldsymbol{F}^*_{\text{motion}}$ and $\boldsymbol{F}^*_{\text{active-force}}$ represent the inputs to the decoupled system. The generalized joint forces $\Gamma$ required to produce the operational forces $F_o$ are

$$\Gamma=J_o^T(\boldsymbol{q})F_o. \qquad (14)$$

With perfect estimates, the resulting closed loop system is described by the following two decoupled subsystems:

$$\Omega\dot{\vartheta}=\Omega\boldsymbol{F}^*_{\text{motion}}; \qquad (15)$$

$$\bar{\Omega}(\dot{\vartheta}+\boldsymbol{F}_s)=\bar{\Omega}(\boldsymbol{F}_{\text{desired}}+\boldsymbol{F}^*_{\text{active-force}}). \qquad (16)$$

The unified motion and force control system is shown in Fig. 4. To further enhance the efficiency of the real-time implementation, the control system is decomposed into two layers—a low rate dynamic parameter evaluation layer, updating the dynamic parameters, and a high rate servo control layer that computes the command vector using the updated dynamic coefficients. This is achieved by factoring the equations of motion into the product of a matrix with coefficients independent of the velocities, and a vector which contains the velocity terms. The matrix of coefficients is then given as a function of the manipulator's configuration. With this separation of the velocity and configuration dependency of the dynamics, the real-time computation of the equations of motion coefficients can be paced by the rate of configuration changes, which is much lower than that of the mechanism dynamics.

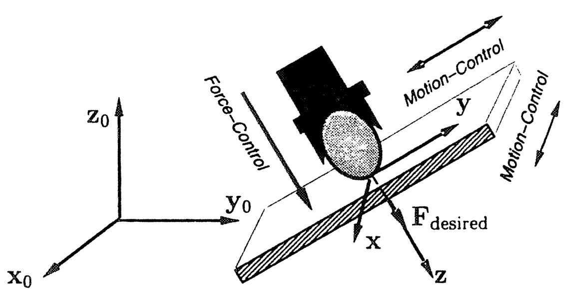


Fig. 3 A constrained motion task

## 5. Redundant Manipulators

A set of operational coordinates, which only describes the end-effector position and orientation, is obviously insufficient to completely specify the configuration of a redundant manipulator. Therefore, the dynamic behavior of the entire system cannot be described by a dynamic model using operational coordinates. Nevertheless, the dynamic behavior of the end-effector itself can still be described, and its equations of motion in operational space can still be established. In fact, the structure of the effector dynamic model has been shown[8],[21] to be identical to that obtained in the case of non-redundant manipulators (given in Eq. ( 3 )). In the redundant case, however, the matrix $\Lambda$ should be interpreted as a "pseudo kinetic energy matrix". As shown below, this matrix is related to the joint space kinetic energy matrix by

$$\Lambda^{-1}(\boldsymbol{q})=J(\boldsymbol{q})A^{-1}(\boldsymbol{q})J^T(\boldsymbol{q}). \qquad (17)$$

The above relationship provides a general expression for the matrix $\Lambda$ that applies to both redundant and non-redundant manipulators. While Eq. ( 3 ) provides a description of the whole system dynamics for non-redundant manipulators, the equation associated with a redundant manipulator only describes the dynamic behavior of its end-effector. In that case, the equation can be thought of as a "projection" of the system's dynamics into the operational space. The remainder of the dynamics will affect joint motions in the null space of the redundant system. This analysis is discussed below.

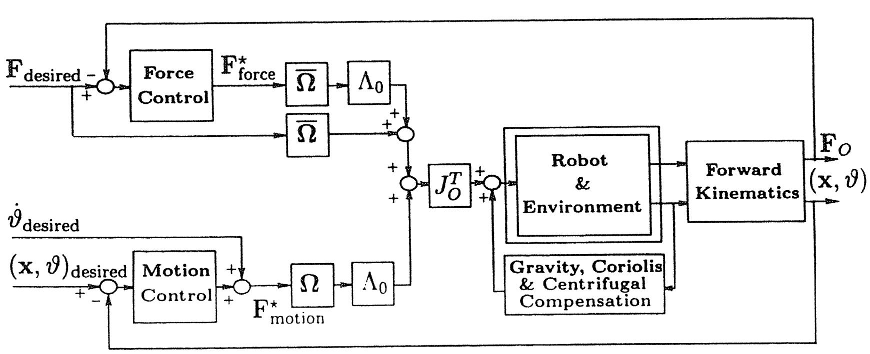


Fig. 4 Unified motion and force control structure

The operational space equations of motion describe the dynamic response of a manipulator to the application of an operational force $\boldsymbol{F}$ at the end-effector. For non-redundant manipulators, the relationship between operational forces, $\boldsymbol{F}$, and joint torques, $\boldsymbol{\Gamma}$ is

$$\boldsymbol{\Gamma}=J^T(\boldsymbol{q})\boldsymbol{F}. \qquad (18)$$

However, this relationship becomes incomplete for redundant manipulators that are in motion. Analysis of the kinematic aspect of redundancy shows that at a given configuration, there is an infinity of elementary displacements of the redundant mechanism that could take place without altering the configuration of the effector. Those displacements correspond to motion in the null space associated with a generalized inverse of the Jacobian matrix.

There is also a null space associated with the transpose of the Jacobian matrix. When the redundant manipulator is not at static equilibrium, there is an infinity of joint torque vectors that could be applied without affecting the resulting forces at the end-effector. These are the joint torques acting within the null space of $J^T(\boldsymbol{q})$. With the addition of null space joint torques, the relationship between end-effector forces and manipulator joint torques takes the following general form

$$\boldsymbol{\Gamma}=J^T(\boldsymbol{q})\boldsymbol{F}+[I-J^T(\boldsymbol{q})J^{\#T}(\boldsymbol{q})]\boldsymbol{\Gamma}_0; \qquad (19)$$

where $\boldsymbol{\Gamma}_0$ is an arbitrary generalized joint torque vector, which will be projected in the null space of $J^T$, and $J^{\#T}$ is a generalized inverse of $J^T$. Clearly, Eq. (19) is dependent on $J^{\#T}$ and there is an infinity of generalized inverses for $J^T$, namely, $\{J^{\#T} \mid J^T=J^TJ^{\#T}J^T\}$. However, only one of these generalized inverses is consistent with the system dynamics. It has been shown[8] that in order for the joint torque vector $\boldsymbol{\Gamma}_0$ to be precluded from producing any dynamic effect at the operational point, it is necessary that

$$J(\boldsymbol{q})A^{-1}(\boldsymbol{q})[I-J^T(\boldsymbol{q})J^{\#T}(\boldsymbol{q})]\boldsymbol{\Gamma}_0=0. \qquad (20)$$

A generalized inverse of $J(\boldsymbol{q})$ satisfying the above constraint is said to be dynamically consistent[15].

**Theorem 1**: (Dynamic Consistency)

A generalized inverse that is consistent with the dynamic constraint of Eq. (20), $\bar{J}(\boldsymbol{q})$, is unique and is given by

$$\bar{J}(\boldsymbol{q})=A^{-1}(\boldsymbol{q})J^T(\boldsymbol{q})\Lambda(\boldsymbol{q}), \qquad (21)$$

The proof is based on a straightforward analysis of Eq. (20).

Notice that $\bar{J}(\boldsymbol{q})$ of Eq. (21) is actually the generalized inverse of the Jacobian matrix corresponding to the solution of $\delta\boldsymbol{x}=J(\boldsymbol{q})\delta\boldsymbol{q}$ that minimizes the manipulator's instantaneous kinetic energy.

### 5.1 Equations of Motion of Redundant Manipulators

The end-effector equations of motion for a redundant manipulator are of the same form as Eq. (3) established for non-redundant manipulators. In this case, however, the inertial properties vary not only with the end-effector configuration, but also with the manipulator posture.

$$\Lambda(\boldsymbol{q})\ddot{\boldsymbol{x}}+\boldsymbol{\mu}(\boldsymbol{q}, \dot{\boldsymbol{q}})+\boldsymbol{p}(\boldsymbol{q})=\boldsymbol{F}; \qquad (22)$$

where

$$\boldsymbol{\mu}(\boldsymbol{q}, \dot{\boldsymbol{q}})=\bar{J}^T(\boldsymbol{q})\boldsymbol{b}(\boldsymbol{q}, \dot{\boldsymbol{q}})-\Lambda(\boldsymbol{q})\dot{J}(\boldsymbol{q})\dot{\boldsymbol{q}}; \qquad (23)$$

$$\boldsymbol{p}(\boldsymbol{q})=\bar{J}^T(\boldsymbol{q})\boldsymbol{g}(\boldsymbol{q}). \qquad (24)$$

Equations (22) provide a description of the dynamic behavior of the end-effector in operational space. These equations are simply the projection of the joint space equations of motion (1), by the dynamically consistent generalized inverse $\bar{J}^T(\boldsymbol{q})$,

$$\bar{J}^T(\boldsymbol{q})[A(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{b}(\boldsymbol{q}, \dot{\boldsymbol{q}})+\boldsymbol{g}(\boldsymbol{q})=\boldsymbol{\Gamma}]$$
$$\Longrightarrow \Lambda(\boldsymbol{q})\ddot{\boldsymbol{x}}+\boldsymbol{\mu}(\boldsymbol{q}, \dot{\boldsymbol{q}})+\boldsymbol{p}(\boldsymbol{q})=\boldsymbol{F}; \qquad (25)$$

### 5.2 Dynamically consistent torque/force relationship

The dynamically consitent relationship between joint torques and operational forces for redundant manipulator systems is

$$\boldsymbol{\Gamma}=J^T(\boldsymbol{q})\boldsymbol{F}+[I-J^T(\boldsymbol{q})\bar{J}^T(\boldsymbol{q})]\boldsymbol{\Gamma}_0; \qquad (26)$$

This relationship provides a decomposition of joint torques into two dynamically decoupled control vectors: joint torques corresponding to forces acting at the end-effector ($J^T\boldsymbol{F}$); and joint torques that only affect internal motions, ($[I-J^T(\boldsymbol{q})\bar{J}^T(\boldsymbol{q})]\boldsymbol{\Gamma}_0$). Using this decomposition, the end-effector can be controlled by operational forces, while internal motions can be independently controlled by joint torques that are guaranteed not to alter the end-effector's dynamic behavior. This relationship is the basis for implementing the dextrous dynamic coordination strategy for macro-/mini-manipulators discussed in Sec. 6.

### 5.3 Singular configurations

A singular configuration is a configuration $\boldsymbol{q}$ at which the end-effector mobility—defined as the rank of the Jacobian matrix—locally decreases. At a singular configuration, the end-effector locally loses the ability to move along or rotate about some direction of the Cartesian space.

Singularity and mobility are characterized by the determinant of the Jacobian matrix for non-redundant manipulators; or by the determinant of the matrix product of the Jacobian and its transpose for redundant mechanisms. This determinant is a function, $s(\boldsymbol{q})$, that vanishes at each of the manipulator singularities. This function can be further developed into a product of terms,

$$s(\boldsymbol{q})=s_1(\boldsymbol{q})\cdot s_2(\boldsymbol{q})\cdot s_3(\boldsymbol{q})\cdots s_{n_s}(\boldsymbol{q}); \qquad (27)$$

each of which corresponds to one of the different types of singularities associated with the kinematic configuration of the mechanism, e.g., alignment of two links or alignment of two joint axes. $n_s$ is the number of different types of singularities. To a singular configuration there corresponds a singular direction. It is in or about this direction that the end-effector presents infinite effective mass or effective inertia. The end-effector movements remain free in the subspace orthogonal to this direction. In reality, the difficulty with singularities extends to some neighborhood around the singular configuration, as illustrated in Fig. 5. The neighborhood of the $i^{\text{th}}$ singularity, $D_{s_i}$, can be defined as

$$D_{s_i}=\{\boldsymbol{q}|\ |s_i(\boldsymbol{q})|\leq\eta_i\}\ ; \qquad (28)$$

where $\eta_i$ is positive. The basic concept in our approach to end-effector control at kinematic singularities can be described as follows:

> In the neighborhood $D_{s_i}$ of a singular configuration $\boldsymbol{q}$, the manipulator is treated as a redundant system in the subspace orthogonal to the singular direction. End-effector motions in that subspace are controlled using the operational space redundant manipulator control. $s_i$ is treated as a new task coordinate. This coordinate is used in the control of end-effector behavior along the singular direction. The control is implemented using the dynamically consistent joint torques acting in the null space associated with the redundancy.

Moving the end-effector to a singular configuration, for instance, is achieved by a control that takes $s_i(\boldsymbol{q})$ to zero. One strategy[21] for moving the end-effector out of a singularity is to control the rate of $s_i(\boldsymbol{q})$. With the two possible assignments of the sign for the desired rate of $s_i(\boldsymbol{q})$, it is possible to select the posture of the manipulator among the two configurations that it can generally take when moving out of a singularity. The rate of $s_i(\boldsymbol{q})$ should be selected according to the desired velocity at the configuration when $|s_i(\boldsymbol{q})|=\eta_i$, in order to achieve a smooth transition when crossing the singularity neighborhood.

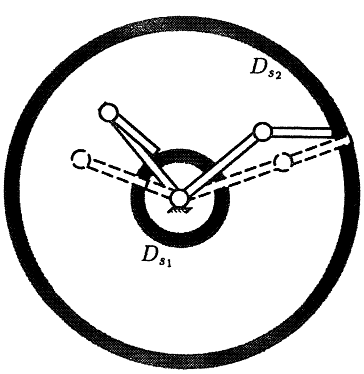


Fig. 5 Kinematic singularities

## 6. Macro-/Mini-Manipulator Systems

A macro-/mini-manipulator can be viewed as the mechanism resulting from the serial combination of two manipulators. As illustrated in Fig. 6a, the manipulator connected to the ground is the macro-manipulator, and the second manipulator, refered to as the mini-manipulator, is the structure formed by the distal set of links that have full freedom to move in the operational space. The macro-manipulator has at least one degree of freedom.

Let $\Lambda_O$ be the pseudo kinetic energy matrix associated with the macro-/mini-manipulator and $\Lambda_{m(O)}$ the operational space kinetic energy matrix associated with the mini-manipulator. Our analysis of macro-/mini-manipulator systems has shown[15] the inertial properties of these systems to possess the following characteristic:

**Theorem 2**: (Reduced Effective Inertia)

The operational space pseudo kinetic energy matrices $\Lambda_O$ (combined mechanism), and $\Lambda_{m(O)}$ (mini-manipulator) satisfy

$$\lambda_k(\Lambda_O)\leq\lambda_k(\Lambda_{m(O)})\ ;\ k=1, 2, \cdots, m\ ; \qquad (29)$$

where $\lambda_k(\cdot)$ denotes the $k^{\text{th}}$ largest eigenvalue of $(\cdot)$, i.e., $\lambda_n(\cdot)\leq\cdots\leq\lambda_1(\cdot)$.

For all directions and configurations, the effective inertia of a macro-/mini-manipulator system (see Fig. 6a), is bounded above by the inertia of the mini-manipulator alone (see Fig. 6b).

A more general statement of Theorem 2 is that the inertial properties of a redundant manipulator are bounded above by the inertial properties of the structure formed by the smallest distal set of degrees of freedom that span the operational space. The equality of the inertial properties in Theorem 2 is obtained for mechanisms that only involve prismatic joints[15].

### 6.1 Dextrous dynamic coordination

The dynamic characteristics of a macro-/mini-manipulator system can be made to be comparable to

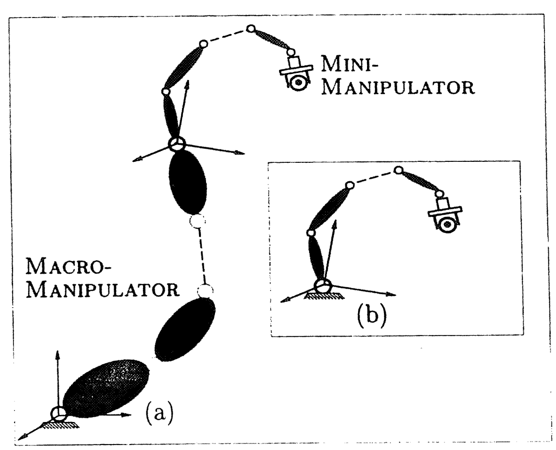


Fig. 6 A macro-/mini-manipulator system

(and, in some cases, better than) those of the mini-manipulator. The basic idea in approaching the control problem associated with coordinating a manipulator and a mini-manipulator system is to treat the manipulator and mini-manipulator as a single redundant system. However, this type of control cannot be directly applied to the macro/mini motion coordination problem. In effect, given the mechanical limits on joint motions of the mini-manipulator, such a controller would rapidly lead to joint saturation of the mini-manipulator degrees of freedom.

The dextrous dynamic coordination we propose is developed within the framework of redundant manipulator control in operational space. It is based on minimizing the deviation from the midrange joint positions of the mini-manipulator. This minimization is achieved using joint torques selected from the dynamically consistent null space of Eq. (26). This will eliminate any effect of the additional forces on the primary task. Let $\bar{q}_i$ and $\underline{q}_i$ be the upper and lower bounds on the $i^{th}$ joint position $q_i$. We construct the potential function

$$V_{\mathrm{Dextrous}}(\boldsymbol{q})=k_d\sum_{i=n_M+1}^{n}\left(q_i-\frac{\bar{q}_i+\underline{q}_i}{2}\right)^2; \tag{30}$$

where $k_d$ is a constant coefficient. The gradient of this function

$$\boldsymbol{\Gamma}_{\mathrm{Dextrous}}=-\nabla V_{\mathrm{Dextrous}}; \tag{31}$$

provides the required attraction[22] to the mid-range joint positions of the mini-manipulator. The interference of these additional torques with the end-effector dynamics is avoided by projecting them into the null space of $J^T(\boldsymbol{q})$. This is

$$\boldsymbol{\Gamma}_{nd}=[I_n-J^T(\boldsymbol{q})\bar{J}^T(\boldsymbol{q})]\boldsymbol{\Gamma}_{\mathrm{Dextrous}}. \tag{32}$$

In addition, joint-limit avoidance can be achieved using an "artificial potential" function[22]. Several other internal motion behaviors have been discussed in the context of controlling free-flying robotic systems[23]. It is essential that the range of motion of the joints associated with the mini-manipulator accommodate the relatively slower dynamic response of the arm. A sufficient margin of motion is required to achieve dextrous dynamic coordination.

## 7. Multi-Effector/Object System

We now consider the problem of object manipulation in a parallel system of $N$ manipulators. The effectors are assumed to be rigidly connected to the manipulated object. The number of degrees of freedom of the parallel system will be denoted by $n_s$.

In this article, we will only discuss the case of a system of $N$ non-redundant manipulators that have all the same number of degrees of freedom, $n$. The end-effectors are also assumed to have the same number of degrees of freedom, $m$ $(m=n)$, as illustrated in Fig. 7. Under these assumptions, the number of degrees of freedom of the parallel system in the planar case $(n=m=3)$ is $n_s=3$. In the spatial case $(n=m=6)$, this number is $n_s=6$. The extension to systems involving redundant manipulators is discussed in Ref. (24).

### 7.1 Augmented object model

To analyze the dynamics of this multi-effector system, we start by selecting the operational point as a fixed point on the manipulated object. Because of the rigid grasp assumption, this point is also fixed with respect to the end-effectors. The number of operational coordinates, $m$, is equal to the number of degrees of freedom, $n_s$, of the system. Therefore, these coordinates form a set of generalized coordinates for the system in any domain of the workspace that excludes kinematic singularities. Thus the kinetic energy of the system is a quadratic form of the generalized operational velocities, $1/2\dot{\boldsymbol{x}}^T\Lambda_{\oplus}(\boldsymbol{x})\dot{\boldsymbol{x}}$. The $m\times m$ kinetic energy matrix $\Lambda_{\oplus}(\boldsymbol{x})$ describes the combined inertial properties of the object and the $N$ manipulators at the operational point. $\Lambda_{\oplus}(\boldsymbol{x})$ can be viewed as the kinetic energy matrix of an augmented object representing the total mass/inertia at the operational point.

Now let $\Lambda_i(\boldsymbol{x})$ be the kinetic energy matrix associated with the $i^{\mathrm{th}}$ unconnected end-effector expressed with respect to the operational point and $\Lambda_{\mathcal{L}}(\boldsymbol{x})$ the kinetic energy matrix associated object itself. It has been shown[24] that

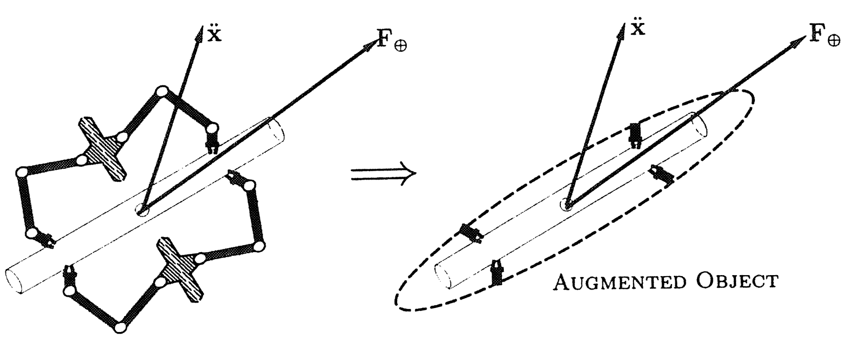


Fig. 7 A multi-arm robot system

**Theorem 3**: (Augmented Object)

The kinetic energy matrix of the augmented object is

$$\Lambda_{\oplus}(\boldsymbol{x})=\Lambda_{\mathcal{L}}(\boldsymbol{x})+\sum_{i=1}^{N}\Lambda_i(\boldsymbol{x}). \tag{33}$$

The use of the additive property of the augmented object's kinetic energy matrix of Theorem 3 allows to obtain the system equations of motion from the equations of motion of the individual manipulators. The augmented object equations of motion are

$$\Lambda_{\oplus}(\boldsymbol{x})\ddot{\boldsymbol{x}}+\boldsymbol{\mu}_{\oplus}(\boldsymbol{x}, \dot{\boldsymbol{x}})+\boldsymbol{p}_{\oplus}(\boldsymbol{x})=\boldsymbol{F}_{\oplus}. \tag{34}$$

The vector, $\boldsymbol{\mu}_{\oplus}(\boldsymbol{x}, \dot{\boldsymbol{x}})$, of centrifugal and Coriolis forces also has the additive property

$$\boldsymbol{\mu}_{\oplus}(\boldsymbol{x}, \dot{\boldsymbol{x}})=\boldsymbol{\mu}_{\mathcal{L}}(\boldsymbol{x}, \dot{\boldsymbol{x}})+\sum_{i=1}^{N}\boldsymbol{\mu}_i(\boldsymbol{x}, \dot{\boldsymbol{x}}); \tag{35}$$

where $\boldsymbol{\mu}_{\mathcal{L}}(\boldsymbol{x}, \dot{\boldsymbol{x}})$ and $\boldsymbol{\mu}_i(\boldsymbol{x}, \dot{\boldsymbol{x}})$ are the vectors of centrifugal and Coriolis forces associated with the object and the $i^{th}$ effector, respectively. Similarly, the gravity vector is

$$\boldsymbol{p}_{\oplus}(\boldsymbol{x})=\boldsymbol{p}_{\mathcal{L}}(\boldsymbol{x})+\sum_{i=1}^{N}\boldsymbol{p}_i(\boldsymbol{x}), \tag{36}$$

where $\boldsymbol{p}_{\mathcal{L}}(\boldsymbol{x})$ and $\boldsymbol{p}_i(\boldsymbol{x})$ are the gravity vectors associated with the object and the $i^{th}$ effector. The generalized operational forces $\boldsymbol{F}_{\oplus}$ are the resultant of the forces produced by each of the $N$ effectors at the operational point.

$$\boldsymbol{F}_{\oplus}=\sum_{i=1}^{N}\boldsymbol{F}_i. \tag{37}$$

The effector's operational forces $\boldsymbol{F}_i$ are generated by the corresponding manipulator actuators. The generalized joint torque vector $\boldsymbol{\Gamma}_i$ corresponding to $\boldsymbol{F}_i$ is given by

$$\boldsymbol{\Gamma}_i=J_i^T(\boldsymbol{q}_i)\boldsymbol{F}_i;$$

where $\boldsymbol{q}_i$ is the vector of joint coordinates associated with the $i^{th}$ manipulator and $J_i^T(\boldsymbol{q}_i)$ is the Jacobian matrix of the $i^{th}$ manipulator computed with respect to the operational point. The dynamic decoupling and motion control of the augmented object in operational space is achieved by selecting a control structure similar to that of a single manipulator,

$$\boldsymbol{F}_{\oplus}=\hat{\Lambda}_{\oplus}(\boldsymbol{x})\boldsymbol{F}^*+\hat{\boldsymbol{\mu}}_{\oplus}(\boldsymbol{x}, \dot{\boldsymbol{x}})+\hat{\boldsymbol{p}}_{\oplus}(\boldsymbol{x}); \tag{38}$$

where, $\hat{\Lambda}_{\oplus}(\boldsymbol{x})$, $\hat{\boldsymbol{\mu}}_{\oplus}(\boldsymbol{x}, \dot{\boldsymbol{x}})$, and $\hat{\boldsymbol{p}}_{\oplus}(\boldsymbol{x})$ represent the estimates of $\Lambda_{\oplus}(\boldsymbol{x})$, $\boldsymbol{\mu}_{\oplus}(\boldsymbol{x}, \dot{\boldsymbol{x}})$, and $\boldsymbol{p}_{\oplus}(\boldsymbol{x})$. With a perfect nonlinear dynamic decoupling, the augmented object of Eq. (34) under the command of Eq. (38) becomes equivalent to a unit mass, unit inertia object, $I_m$, moving in the $m$-dimensional space,

$$I_m\ddot{\boldsymbol{x}}=\boldsymbol{F}^*. \tag{39}$$

Here, $\boldsymbol{F}^*$ is the input to the decoupled system. The control structure for constrained motion and active force control operations is similar to that of a single manipulator.

The control structure in Eq. (38) provides the net force $\boldsymbol{F}_{\oplus}$ to be applied to the augmented object at the operational point for a given control input, $\boldsymbol{F}^*$. Due to the actuator redundancy of multi-effector systems, there is an infinity of joint-torque vectors that correspond to this force.

In tasks involving large and heavy objects, a useful criterion for force distribution is the minimization of total actuator effort[24]. However, dextrous manipulation requires accurate control of internal forces. This problem has received wide attention and algorithms for internal force minimization[25] and grasp stability[26] have been developed. To address the problem of internal force characterization, a physically-based virtual linkage model has been proposed[27] to describe and control internal forces and moments in multi-grasp tasks.

## 8. Conclusion

In this review, we have presented the various models and methodologies developed in the operational space framework. The basis of this framework is a model that describes the dynamics of a manipulator in terms of its behavior at the end-effector. This model provides the foundation for a unified approach to task-level motion and force control.

Discussing the extension of this approach to redundant manipulator systems, we have presented the model that describes dynamic behavior at the end-effector of a redundant manipulator. We have also presented the dynamically consistent force/torque relationship for these systems. With this relationship, joint torques are decomposed into two dynamically decoupled control vectors: joint torques corresponding to forces acting at the end-effector; and joint torques that only affect internal motions. Using this decomposition, the end-effector can be independently controlled by operational forces, while internal motions can be controlled by joint torques that are guaranteed not to alter the end-effector's dynamic behavior. In addition to the control of redundant manipulator, these models have been the basis for a new strategy for dealing with kinematic singularities. With this strategy, a manipulator at a singular configuration is treated as a redundant system in the subspace orthogonal to the singular direction.

Our analysis of inertial properties for macro-/mini-manipulator systems has shown that, for all directions and configurations, the effective mass/inertia of a macro-/mini-manipulator is less than or equal to the inertia associated with the mini-manipulator structure, considered alone. To allow the mini-structure's high bandwidth to be fully utilized in wide range operations, we have proposed a dextrous dynamic coordination strategy which uses the system's internal motions to minimize deviation from the midrange joint positions of the mini-manipulator.

Effective implementation of this strategy relies on preventing any effects of the internal motion from influencing the primary end-effector task. This is achieved by using the dynamically consistent relationship between joint torques and end-effector forces.

Analyzing the inertial properties of multi-arm robot systems, we have presented an important additive property of parallel structures. It has been shown that the inertial properties perceived at the manipulated object are given by the sum of the inertial properties associated with each individual manipulator and the inertial properties of the unconstrained object, all expressed with respect to the same operational point. Centrifugal, Coriolis, and gravity forces have also been shown to possess this additive property. Combining the dynamics of the individual manipulators and object, we have proposed the augmented object as a model of the dynamics at the operational point for the multi-arm robot system.

By providing task-level models of robot dynamics, the operational space framework answers many of the deficiencies associated with joint space formulations. It is important to emphasize the fact that operational space implementations rely to a large extent on the robot's ability to achieve effective control of joint torques. This capability is, in fact, a key requirement for any dynamic control implementation —including joint space dynamic control implementations. With today's robots, this ability is considerably restricted by the nonlinearities and friction inherent in their actuator-transmission systems. However, recent trends and current developments suggest that the new generation of robot system can be expected to provide improved joint torque control capabilities that would clearly enable more effective implementations of advanced dynamic control techniques.

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