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Design of Macro/Mini Manipulators for Optimal Dynamic Performance

Alan Bowling and Oussama Khatib

Robotics Laboratory
Department of Computer Science
Stanford University, Stanford, CA 94086

Abstract

This article investigates the problem of redundant manipulator design for optimal dynamic performance as applied to the design of macro/mini structures. The dynamic performance of a manipulator is characterized by the inertial and acceleration properties of the end-effector. However, for redundant manipulators the characteristics of motions in the end effector null space must also be considered. This article presents a methodology for analyzing the performance requirements for the null space motions. The analysis results in a decomposition of the overall design problem into a set of smaller subproblems. Optimization techniques are then used to determine the design parameters which improve manipulator dynamic performance. The decomposition greatly reduces the search space of the overall optimization. Here this methodology is presented along with the models and measures upon which it is based. The approach is illustrated in the selection of design parameters for a simple six-degree-of-freedom planar mechanism.

1 Introduction

The study of manipulator dynamic performance involves issues dealing with how quickly the system responds to actuator commands and how well it reacts to contact forces and moments. Our previous work in this area focused on the design of non-redundant manipulators [6] and resulted in a set of characterizations and measures which were used to formulate the optimization problem. Here this approach is extended to redundant mechanisms; specifically macro/mini manipulator systems having twelve or fewer degrees of freedom (DOF). In this article the mini manipulator is the structure comprised of the smallest distal set of degrees of freedom which has the same number of DOF as the end effector. The remaining links form the macro structure of the manipulator.

There is a large body of work addressing redundant manipulator design. The general approach has been to determine measures which quantify desired characteristics and then to base the design process on improving those measures. Many studies base their design methodology on analysis of the manipulator jacobian [1, 7, 8, 9, 10, 12]. Analysis of the jacobian is useful for kinematic synthesis of the mechanism and considerations of force control. However, the jacobian does not contain information about the inertial properties of the mechanism which must be overcome in order to produce motion. Other studies include dynamic effects in their analyses, [2, 3, 5, 11]. However, these studies did not involve redundant mechanisms and/or did not adequately deal with the problem of inhomogeneity between linear and angular motion which are encountered when considering manipulators with greater than three degrees of freedom.

Our previous design studies were based on analysis of end effector inertial and acceleration characteristics. The inertial properties as perceived at the end effector are important because they determine the natural behavior of the mechanism when coming into contact with the environment. End effector acceleration properties, also affected by the inertial properties, determine its responsiveness to commands from a controller. In these studies separate measures and characterizations for the properties associated with end effector linear and angular motions were proposed. However, in redundant manipulator design it is not enough to only consider end effector properties. The effect of the redundant degrees of freedom on the overall mechanism performance must also be analyzed. Careful consideration of the acceleration properties of a manipulator reveals the fact that, in addition to end effector performance, redundant systems must also exhibit adequate dynamic performance in the end effector null space.

This article presents an approach for analyzing manipulator null space motions using the measures from our previous studies. In performing this analysis it becomes clear that this approach quickly leads to a decomposition of the design problem into smaller sub-

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problems. This results in a much smaller search space for the optimization of the overall design. The decomposition relies on the fact that the manipulator inertial and acceleration properties can be described in terms of the properties of the mini manipulator. The overall goal of the optimization can be stated in two parts; (a) to obtain a mechanism with the smallest, most isotropic inertial properties and the largest, most isotropic acceleration capability at its end effector, over the work space, and (b) to obtain a mechanism with enough null space performance to support and enhance the end effector acceleration capability.

In the following sections an overall approach to redundant manipulator design is presented. Here the focus is design of systems having twelve or fewer degrees of freedom. However, this methodology can be extended to higher DOF systems. A brief discussion of the measures and characterizations of inertial and acceleration properties is then presented. Finally the results from the application of the process to the design of a six DOF planar manipulator are presented.

2 Overview

The first step in this analysis is to subdivide the redundant mechanism into two non-redundant subsystems; the macro and mini manipulators. This is done for the purpose of analyzing and characterizing the null space motions in a way that is physically meaningful. Next the interactions between the two subsystems are considered. Examination of these interactions quickly leads to the conclusion that the overall design problem can be reduced into separate subproblems. Furthermore, the design of the macro manipulator can be based completely on the mini manipulator design. The resulting process can be summarized as the design of a mini manipulator for a desired performance, assuming that the macro is fixed, followed by the design of a macro manipulator to complement and enhance the performance of the mini.

In order for the two subsystems to complement each other, the desired characteristics for the dynamic interactions between them must be described and quantified. Our desired interactions can be stated as two design constraints:

1. The macro manipulator must be able to stabilize itself against all mini manipulator motion
2. The mini manipulator must be able to stabilize itself against all macro manipulator motion.

The first constraint implies that the macro should be capable of holding itself fixed against motion of the mini. This constraint is analyzed by considering the reaction forces on the macro caused by the mini manipulator motions, as shown in Figure 1 where F_r represents the reaction forces and moments. The macro should be designed to cancel the reaction forces produced by the mini.

The second constraint means that the mini must be capable of holding its end effector fixed in the workspace while the macro performs all possible motions. This constraint maps directly to a requirement on null space performance. The constraint is analyzed by developing equations describing the motion of the overall end effector in terms of the motion of the two subsystem's end effectors.

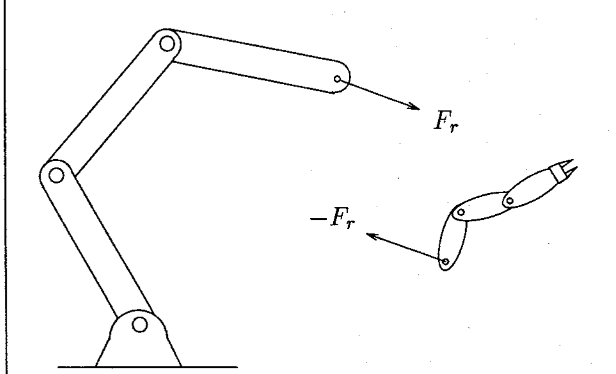

Figure 1: Macro and Mini Manipulators.

The results of satisfying these two constraints are an upper and lower bound on the macro performance requirements. The first constraint gives a lower bound on the macro performance while the second yields an upper bound. A macro designed outside of these bounds is considered underdesigned or overdesigned.

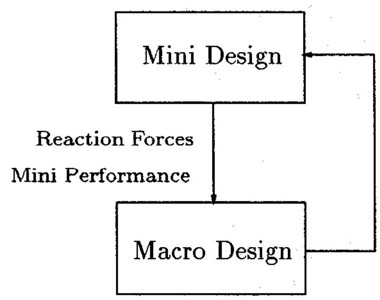

Figure 2: Overall Optimization.

Performance specifications refer to the manipulator's acceleration capability. However, optimizing the inertial properties as perceived at the end effector is also a goal. Analyses of inertial properties for macro/mini manipulator systems have shown that for all directions and configurations, the effective mass/inertia of a macro/mini manipulator is smaller than or equal to the mass/inertia associated with the mini manipulator structure considered alone [4]. This fact supports the proposed decomposition. Thus the main concern with respect to inertial properties is in making the mini properties as small and isotropic as possible in its workspace.

The overall design process is shown in Figure 2. Here the optimization of the mini is performed first. The maximum reaction forces and acceleration capability requirements are determined from the mini design and are passed to the macro design stage. The results from the macro may suggest a reconsideration of the mini design.

3 Measures and Characterizations

Before discussing the details of the analysis of the interactions occuring between the two subsystems it is necessary to briefly present the models used to characterize inertial and acceleration properties.

3.1 Inertial Properties

The inertial properties as perceived at the end-effector are described by the pseudo-kinetic energy matrix $\Lambda(\mathbf{q})$,

$$\Lambda^{-1}(\mathbf{q}) = J(\mathbf{q})A^{-1}(\mathbf{q})J^{T}(\mathbf{q}) \qquad (1)$$

where $A(\mathbf{q})$ is the joint space kinetic energy matrix, $\mathbf{q}$ is the vector of n joint coordinates, and the Jacobian, $J(\mathbf{q})$, is defined as,

$$\boldsymbol{\vartheta} \triangleq \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} = J(\mathbf{q})\dot{\mathbf{q}}, \qquad (2)$$

where $\boldsymbol{v}$ and $\boldsymbol{\omega}$ are the end-effector linear and angular velocities.

Analysis of the matrix $\Lambda(\mathbf{q})$ is based on a decomposition of the Jacobian matrix into its linear and angular sub-matrices:

$$J(\mathbf{q}) = \begin{bmatrix} J_v(\mathbf{q}) \\ J_\omega(\mathbf{q}) \end{bmatrix} \qquad (3)$$

where the matrix $J_v(\mathbf{q})$ transforms joint velocities into end-effector linear velocities and $J_\omega(\mathbf{q})$ does likewise for end-effector angular velocities. From these relationships the effective mass and inertia properties are described by the matrices, Λ_v and Λ_ω;

$$\Lambda_v = (J_v(\mathbf{q})A^{-1}(\mathbf{q})J_v^T(\mathbf{q}))^{-1} \qquad (4)$$

and

$$\Lambda_\omega = (J_\omega(\mathbf{q})A^{-1}(\mathbf{q})J_\omega^T(\mathbf{q}))^{-1}. \qquad (5)$$

More details on the characterization of inertial properties can be found in [4] and [6]. The inertial properties can be represented as a belted ellipsoid, for example Figure 3.

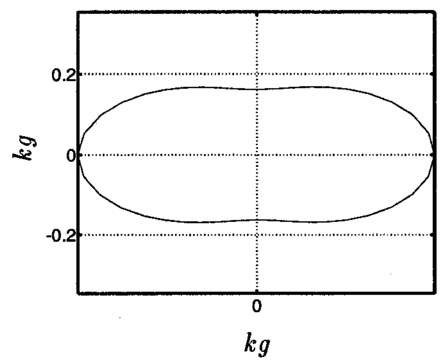

Figure 3: Mass Belted Ellipsoid.

3.2 Acceleration Capability

End-effector acceleration capability is analyzed by determining the isotropic linear and angular accelerations; the largest amount of acceleration achievable at the end effector in or about every direction. The effect of end effector velocities on acceleration capability is also analyzed by modeling them as isotropic velocities. Note that the following development is valid only for non-redundant manipulators.

First consider the bounds on the actuator's torque capability,

$$-\boldsymbol{\Upsilon}_{bound} \leq \boldsymbol{\Upsilon} \leq \boldsymbol{\Upsilon}_{bound}. \qquad (6)$$

End effector dynamic behavior is described by,

$$\Lambda\dot{\boldsymbol{\vartheta}} + \boldsymbol{\mu} + \mathbf{p} = \mathbf{F} \qquad (7)$$

and

$$\boldsymbol{\Upsilon} = \mathcal{T}^{-1}J^{T}\mathbf{F} \qquad (8)$$

where $\boldsymbol{\mu}$, $\mathbf{p}$, $\mathbf{F}$, and $\mathcal{T}$ are respectively the centrifugal and Coriolis force vector, gravity force vector, generalized force vector acting in operational space, and transformation between joint torques and actuator torques. Using the above equations the following relationship is obtained,

$$\boldsymbol{\Upsilon}_{lower} \leq E_v\dot{\boldsymbol{v}} + E_\omega\dot{\boldsymbol{\omega}} + \bar{\mathbf{b}} \leq \boldsymbol{\Upsilon}_{upper} \qquad (9)$$

where

$$[E_v\ E_\omega] = NT^{-1}J^T\Lambda \qquad \Upsilon_{upper} = 1 - NT^{-1}J^T\mathbf{p}$$
$$\bar{\mathbf{b}} = NT^{-1}J^T\mu \qquad \Upsilon_{lower} = -1 - NT^{-1}J^T\mathbf{p}. \tag{10}$$

In obtaining this equation the bounds, Υ_{bound}, have been normalized using a diagonal matrix N with elements $N_{ii} = \frac{1}{\Upsilon_{bound_i}}$.

Equation (9) can be used to derive another set of equations which describes the relationships between end effector isotropic linear and angular, accelerations and velocities; $||\dot{v}||$, $||\dot{\omega}||$, $||v||$, and $||\omega||$. These equations have the form,

$$\mathcal{A}(\mathbf{q})\begin{bmatrix} ||\dot{v}|| \\ ||\dot{\omega}|| \end{bmatrix} + \mathcal{C}(||v||, ||\omega||, \mathbf{q}) = \begin{bmatrix} \Upsilon_{upper} \\ \Upsilon_{lower} \end{bmatrix} \tag{11}$$

where $\mathcal{A}(\mathbf{q})$ is a matrix of coefficients and $\mathcal{C}(||v||, ||\omega||, \mathbf{q})$ is a vector where each element is a quadratic form in $||v||$ and $||\omega||$.

The four-dimensional relationship represented in equation (11) can be visualized as a hypersurface in four-dimensions. We are mainly concerned with the accelerations at zero end effector velocity, described by the first term on the left side of (11). The relationship between the isotropic accelerations is represented as a piecewise linear curve, Figure 4.

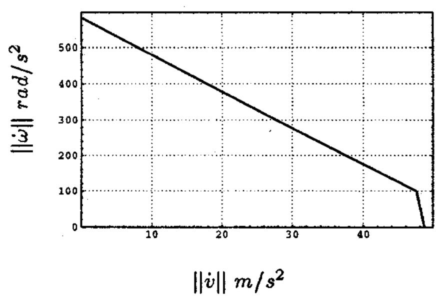

Figure 4: Isotropic Curve.

4 Analysis of Interactions

In this section the analyses required to satisfy the two design constraints discussed in section 2 are presented. The first constraint requires determination of the reaction forces produced by mini manipulator motion. These forces appear in the macro manipulator operational space equations of motion as,

$$\Lambda_{mac}\dot{\vartheta}_{mac} + \mu_{mac} + \mathbf{p}_{mac} = \mathbf{F}_{mac} - \mathbf{F}_r \tag{12}$$

where $\mathbf{F}_r$ represents the reaction forces from the mini. Since the macro must remain motionless, by the first constraint, a lower bound on macro performance requirements can be found from equation (12) as,

$$\Upsilon_r = T^{-1}J^T(\mathbf{F}_r + \mathbf{p}_{mac}). \tag{13}$$

The bound is determined by finding the macro actuator torques, Υ_r, required to cancel the maximum reaction forces from the mini workspace and the maximum gravitational forces from the macro workspace. The achievable isotropic accelerations from this set of actuators represents the lower bound on macro acceleration requirements.

In order to satisfy the second constraint the motions of the overall end effector are described in terms of the motion of the end effectors of the two subsystems. Again, the focus is on the acceleration properties at zero end effector velocities. The equations describing these accelerations are

$$\dot{v} = \dot{v}_{mac} + \dot{v}_{min} + \dot{\omega}_{mac} \times \mathbf{r} \tag{14}$$
$$\dot{\omega} = \dot{\omega}_{mac} + \dot{\omega}_{min} \tag{15}$$

where $\mathbf{r}$ is the position vector from the tip of the end effector of the macro to the end effector of the mini. The subscripts *mac* and *min* indicate the relation to the macro and mini structures, respectively. All quantities in the above equation are assumed to be expressed in the same reference frame. The second constraint states that $\dot{v} = \dot{\omega} = 0$, which leads to;

$$\dot{v}^T_{min}\dot{v}_{min} = \dot{v}^T_{mac}\dot{v}_{mac} + 2\dot{v}^T_{mac}\,\hat{\mathbf{r}}\,\dot{\omega}_{mac} - \dot{\omega}^T_{mac}\,\hat{\mathbf{r}}\,\hat{\mathbf{r}}\,\dot{\omega}_{mac} \tag{16}$$
$$\dot{\omega}^T_{min}\dot{\omega}_{min} = \dot{\omega}^T_{mac}\dot{\omega}_{mac} \tag{17}$$

where if $\mathbf{r} = [\ r_1\ r_2\ r_3\ ]^T$ then

$$\hat{\mathbf{r}} = \begin{bmatrix} 0 & r_3 & -r_2 \\ -r_3 & 0 & r_1 \\ r_2 & -r_1 & 0 \end{bmatrix}. \tag{18}$$

In our characterization, accelerations are described in terms of isotropic accelerations. Thus we need to determine from equations (16) and (17) the largest isotropic values of the mini linear and angular acceleration that correspond to the isotropic macro accelerations. This problem can be expressed as a constrained maximization problem which can be solved using the method of Lagrange multipliers. In order to satisfy the second constraint the isotropic accelerations of the mini must be larger than or equal to the largest values of the right hand sides of equations (16) and (17). Omitting the details, the results of this solution are;

$$||\dot{v}_{min}|| \geq ||\dot{v}_{mac}|| + ||\mathbf{r}||\,||\dot{\omega}_{mac}|| \tag{19}$$
$$||\dot{\omega}_{min}|| \geq ||\dot{\omega}_{mac}||. \tag{20}$$

The macro performance requirements are derived from the mini performance using the above two equations. However, note that the dynamic analysis of the mini and macro from equation (12), shows that in order to provide the performance described in equations (19) and (20), the macro must first overcome the reaction forces due to motion of the mini. Thus the performance specified by equations (19) and (20) is considered to be in addition to the acceleration required to overcome the accelerations associated with the reaction forces produced by the mini motion.

The maximum isotropic magnitude of the overall end effector accelerations can be found as,

$$||\dot{v}|| \leq ||\dot{v}_{mac}|| + ||\dot{v}_{min}|| + ||\mathbf{r}|| \, ||\dot{\omega}_{mac}|| \quad (21)$$

$$||\dot{\omega}|| \leq ||\dot{\omega}_{mac}|| + ||\dot{\omega}_{min}||. \quad (22)$$

Combining equations (19), (20), (21), and (22) shows that the acceleration capability for the end effector of the overall mechanism is upper bounded by twice the acceleration performance of the mini manipulator

$$||\dot{v}|| \leq 2 \, ||\dot{v}_{min}|| \quad (23)$$

$$||\dot{\omega}|| \leq 2 \, ||\dot{\omega}_{min}||. \quad (24)$$

5 Application

This methodology is used in the design of the six DOF planar mechanism shown in Figure 1. The mechanism is decomposed into two non-redundant three DOF subsystems. The results shown in Figures 5 through 8 represent an average of the properties over a set of configurations in the workspace of the mini and macro.

The mini manipulator is optimized first. The design parameters to be optimized are the lengths of each link. The effective inertia of the final design is less than half that of the initial design; the effective inertia is 6.7×10^{-4} kgm^2 for the initial design and 2.9×10^{-4} kgm^2 for the final design.

The mass properties for the macro and mini are shown in Figure 5. The initial and final designs are represented respectively as dashed and solid lines. This figure shows that the effective mass of the final design are approximately twice as good as the initial. Both the initial and final designs have the same overall extension of 45 cm and use the same actuators. However, in the initial design all three links have equal length, 15 cm, while in the final design the base link has a length of 24 cm and the remaining two links each have a length of 10 cm.

The acceleration properties for the mini manipulator are shown in Figure 6. Again the dashed and solid curves represent respectively the initial and final designs. The figure shows a small improvement in the isotropic linear acceleration while the isotropic angular accelerations improve by 1.5 times along the coordinate axes.

Given the mini design, the maximum reaction forces and macro performance requirements can be derived. As stated earlier the overall mechanism's inertial properties are upper bounded by the mini inertial properties. In addition, the overall optimization goal is to keep the macro inertial properties as small as possible.

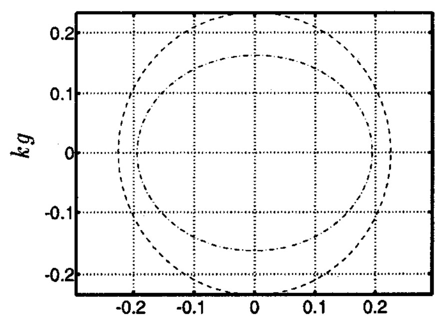

Figure 5: Mini Mass.

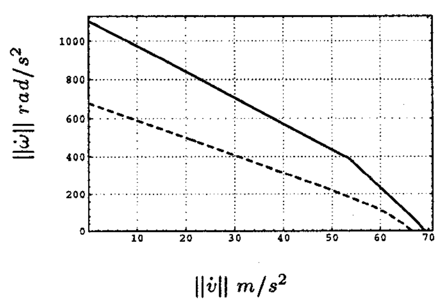

Figure 6: Mini Performance.

The average acceleration properties for the initial macro design are shown in Figure 7. In this figure the upper and lower bounds on macro performance are shown in dashed lines. Again the lower bound is found from the largest mini reaction forces and the upper bound is determined from the mini performance.

It is desirable for the macro acceleration performance, represented by the solid line, to encompass as much of the region between the bounds as possible. However, the behavior of the mechanism is only guaranteed for the region between the upper and lower bounds. The initial design of the macro is close to the upper bound however it loses some performance in the region near the maximum isotropic linear acceleration.

Figure 8 shows that the final design of the macro completely encompasses the region between the performance bounds. In this case the performance of the overall mechanism is considered to be twice that of the mini. Note that the final macro linear acceleration capacity increased over the initial by roughly a factor of 1.4. The initial and final designs have almost the same total extension, 135 *cm*. In the initial design the links have the same length, 45 *cm*, while in the final design the last link has a length of 14 *cm* and the two other links each have length 60 *cm*.

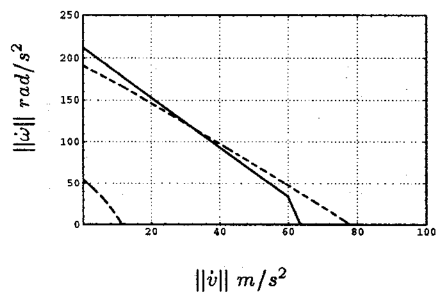

Figure 7: Macro Initial Performance.

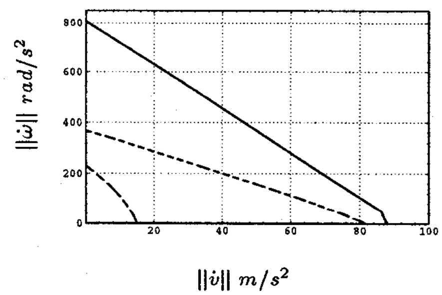

Figure 8: Macro Final Performance.

6 Conclusion

We have presented a general methodology for the design of redundant manipulators for high dynamic performance. This methodology provides a modular decomposition of the design process into separate stages. This greatly reduces the search space for redundant manipulator design optimization, since each of these stages depends on an independent subset of the design parameters. The design process has been illustrated on a simple six DOF planar manipulator. These results can be easily extended to systems involving cascades of macro/mini structures.

Acknowledgements

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