Manipulator Control at Kinematic Singularities: A Dynamically Consistent Strategy

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Abstract

This paper presents a general strategy for manipulator control at kinematic singularities. When a manipulator is in the neighborhood of singular configurations, it is treated as a redundant mechanism in the subspace orthogonal to the singular directions of the end-effector. Control in this subspace is based on operational forces, while null space joint torques are used to deal with the control in the singular directions. Decoupled behavior is guaranteed by using the dynamically consistent *force/torque relationship. Two different types of kinematic singularities are identified and strategies dealing with these singularities are developed. Experimental results of the implementation of this approach on a PUMA 560 manipulator are presented.*

1 Introduction

The difficulty with joint space control techniques lies in the discrepancy between the space where robot tasks are specified and the space in which the control is taking place. By its very nature, joint space control calls for transformations whereby joint space descriptions are obtained from the robot task specifications.

The joint space task transformation problem is exacerbated for mechanisms with redundancy or at kinematic singularities. The typical approach involves the use of pseudo or generalized inverses to solve an underconstrained or degenerate system of linear equations, while optimizing some given criterion [3, 4]. Other inverses with improved performance also have been investigated, e.g., the singularity robust inverse [1, 8].

In addition, dealing with dynamics is essential for achieving higher performance. In free motion, dynamic effects increase with the range of motion, speed, and acceleration at which a robot is operating. In part mating operations, they also increase with the rigidity of the mating object. Furthermore, when controlling end-effector motion with simultaneous control of contact forces in the orthogonal subspace, inertial coupling can significantly affect performance. These effects must be taken into account to achieve higher performance.

The limitations of joint space control techniques, especially in constrained motion tasks, have motivated alternative approaches for dealing with task-level dynamics and control. The *operational space formulation*, which falls within this line of research, has been driven by the need to develop mathematical models for the description, analysis, and control of robot dynamics with respect to task behavior.

In this framework, the control of redundant manipulators relies on two basic models: a *task-level dynamic model* obtained by projecting the manipulator dynamics into the operational space [5], and the *dynamically consistent* force/torque relationship that provides decoupled control of joint motions in the null space associated with the redundant mechanism [6]. These two models are the bases for implementing the control strategy for kinematic singularities. At singular configurations, a manipulator is treated as a redundant mechanism in the subspace orthogonal to the singular directions.

In this paper, we propose a classification of kinematic singularities from control perspective and present a general strategy for manipulator control at kinematic singularities. The effectiveness of this approach is experimentally demonstrated on a PUMA 560 manipulator.

2 Kinematic Singularities

A *singular configuration* is a configuration $\mathbf{q}$ at which the end-effector mobility – defined as the rank

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of the Jacobian matrix – locally decreases. At a singular configuration, the end-effector locally loses the ability to move along or rotate about some direction in Cartesian space.

Singularities and mobility are characterized by the determinant of the Jacobian matrix for non-redundant manipulators; or by the determinant of the matrix product of the Jacobian and its transpose for redundant mechanisms. This determinant is a function, $s(\mathbf{q})$, that vanishes at each of the manipulator singularities. This function can be developed into a product of terms,

$$s(\mathbf{q}) = s_1(\mathbf{q}) \cdot s_2(\mathbf{q}) \cdot s_3(\mathbf{q}) \dots s_{n_s}(\mathbf{q}); \qquad (1)$$

where each term corresponds to one of the different singularities associated with the mechanism. Here, n_s is the number of different singularities. A singular configuration always has a corresponding *singular direction.* It is in or about this direction that the end-effector presents infinite effective mass or effective inertia. The end-effector movements remain free in the subspace orthogonal to this direction. In reality, the difficulty with singularities extends to some neighborhood around the singular configuration. The neighborhood of the i^{th} singularity, $\mathcal{D}_{s_i}$, can be defined as

$$\mathcal{D}_{s_i} = \{\mathbf{q} \mid |s_i(\mathbf{q})| \leq \eta_i\}; \qquad (2)$$

where η_i is positive.

In the neighborhood $\mathcal{D}_{s_i}$ of a singular configuration $\mathbf{q}$, the manipulator is treated as a redundant mechanism in the subspace[1] orthogonal to the singular direction. End-effector motions in that subspace are controlled using the operational space redundant manipulator control, while null space joint torques are used to deal with the control in the singular directions. The use of the *dynamically consistent* force/torque relationship guarantees decoupled behavior between end-effector control and null space control.

3 Dynamic Consistency

In order to achieve dynamically consistent behavior of the redundant manipulator, it is necessary to decouple the motion in the null space from the motion in the operational space. In other words, the selection of null space motion control torques from the null space should not generate any acceleration at the end-effector in the operational space.

This can be achieved using the dynamically consistent relationship between operational forces and joint torques for redundant manipulators [6]

$$\mathbf{\Gamma} = J^T(\mathbf{q})\mathbf{F} + \left[I - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})\right]\mathbf{\Gamma}_0; \qquad (3)$$

where $\overline{J}$ is the dynamically consistent generalized inverse of the Jacobian matrix given as

$$\overline{J}(\mathbf{q}) = A^{-1}(\mathbf{q})J^T(\mathbf{q})\Lambda(\mathbf{q}). \qquad (4)$$

$\mathbf{q}$ is the vector of joint coordinates, $A(\mathbf{q})$ is the joint space kinetic energy matrix, $J(\mathbf{q})$ is the Jacobian matrix, and $\Lambda(\mathbf{q})$ is the operational space kinetic energy matrix. $\overline{J}$ has been shown to be unique [6].

The above relationship provides a decomposition of joint torques into two dynamically decoupled control vectors: joint torques corresponding to forces acting at the end effector, $J^T\mathbf{F}$; and joint torques that only affect null space joint motions, $[I - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\mathbf{\Gamma}_0$. $\mathbf{F}$ is the operational space control forces acting on the end-effector; $[I - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]$ is the projection onto the null space; and $\mathbf{\Gamma}_0$ is the joint control torques for desired motions in the null space.

An additional task to be carried out using the null space can be realized by constructing a potential function, $V_0(\mathbf{q})$, whose minimum corresponds to the desired task. This is accomplished by selecting

$$\mathbf{\Gamma}_0 = -A(\mathbf{q})\left[\nabla V_0 + k_{vq}\dot{\mathbf{q}}\right]; \qquad (5)$$

where $-k_{vq}\dot{\mathbf{q}}$ corresponds to the dissipative torques needed to provide asymptotic stabilization of the mechanism in the null space and $-[\nabla V_0 + k_{vq}\dot{\mathbf{q}}]$ is weighted by $A(\mathbf{q})$ to account for the manipulator dynamics. The interference from the additional torques on the end-effector is eliminated by projecting this vector onto the null space.

Using this decomposition, the end effector can be controlled by operational forces, while motions in the null space can be independently controlled by joint torques that are guaranteed not to alter the end effector's dynamic behavior.

3.1 Simulation Example

The impact of the dynamically consistent control decomposition is illustrated on the 3R-planar manipulator shown in Figure 1 and Figure 2. This manipulator is treated as a redundant mechanism with respect to the task of positioning the end-effector.

The goal here is to maintain the end-effector position, while letting the manipulator move in the null space. The end-effector position is controlled by operational forces. An oscillatory motion in the null space is produced by the application, in the null space, of

[1] a subspace of the end-effector operational space.

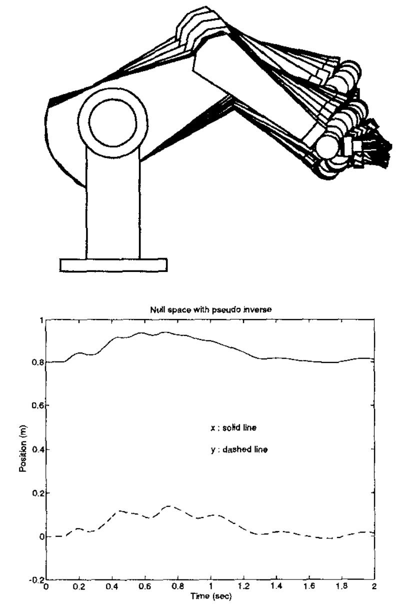

Figure 1: Null Space Motion with Pseudo Inverse

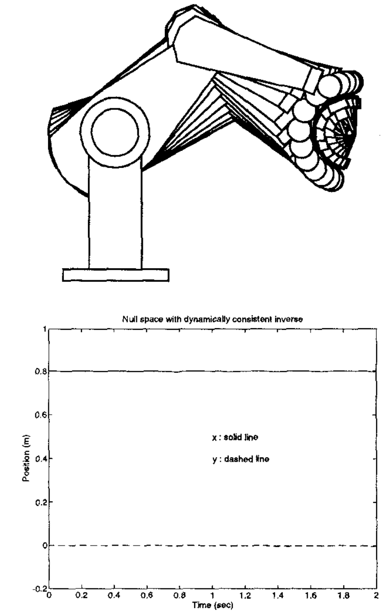

Figure 2: Null Space Motion with Dynamically Consistent Inverse

the negative gradient of an attractive potential without any dissipative forces, i.e., $\mathbf{\Gamma}_0 = -A(\mathrm{q})\nabla V_0$.

Two different generalized inverses are used to construct the projection of $\mathbf{\Gamma}_0$ onto the null space: the Moore-Penrose or pseudo inverse, $J^+ = J^T(JJ^T)^{-1}$, and the dynamically consistent inverse, $\overline{J}$ from Equation 4. The simulation results are shown in Figure 1 and Figure 2.

As expected, with the dynamically consistent inverse (see Figure 2) the motion in the null space does not affect the end-effector position, while large coupling forces are produced at the end-effector when the pseudo inverse is used (see Figure 1). Doty [2] has extensively discussed the difficulty with pseudo inverses and observed the need for dynamically weighted generalized inverses.

4 Control Strategy at Singularities

The dynamically consistent torques/forces decomposition of a non-redundant mechanism is done as follows: First, in the neighborhood of singular configurations, singular directions and the associated singular frames are identified. A *singular frame* is a frame in which one of the axes is aligned with the singular direction. Next, the Jacobian matrix of the non-redundant mechanism is rotated into the singular frame and the rows corresponding to singular directions are eliminated. This new Jacobian matrix corresponds to the redundant mechanism with respect to end-effector motion in the subspace orthogonal to the singular directions. The dynamically consistent inverse of this Jacobian matrix from Equation 4 is used to construct the projection onto the null space and this projection is used to control motions in the null space. End-effector motions in the subspace orthogonal to the singular directions are controlled using the operational space redundant manipulator control [5].

4.1 Two Types of Singularities

In previous work [7], singularities have been characterized in terms of the internal freedom of motion a manipulator has at a singular configuration while its end-effector remains fixed.

However, for control purposes, we separate singularities into two groups based on the control characteristics of their null spaces. Type 1 singularities are those at which the end-effector can be controlled in the singular directions using motion in the null space. Type 2 singularities are those at which motion in the null space only affects the singular direction.

Figure 3 shows the three basic singularities in a PUMA 560: *elbow lock, wrist lock, and overhead lock* from left to right. Elbow lock is a Type 1 singularity and the other two are Type 2 singularities.

Joint motions in the null space associated with a Type 1 singularity results in a motion of the end-effector in the singular direction. However, joint motions in the null space associated with a Type 2 singularity results only in a change of the singular direction through internal joint motions. Two different strategies are developed according to these two types of singularities.

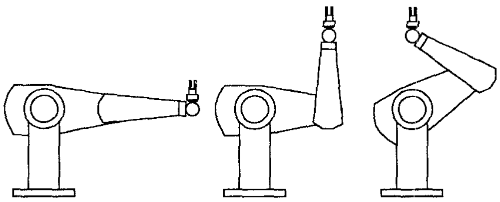

Figure 3: Three Basic Singular Configurations in PUMA 560

4.2 Type 1 Singularity

The end-effector motion in the singular direction is controlled directly through the associated null space. $s_i(\mathbf{q})$ from Equation 1 is treated as a new task coordinate. A potential function of this coordinate is used in the control of end-effector behavior along the singular direction. Using this potential function, the projection of the torques from Equation 5 onto the null space affects the end-effector motion only along the singular direction.

Moving the end-effector to a singular configuration is achieved by a control that takes $s_i(\mathbf{q})$ to zero. When moving out of a singularity, the control of the end-effector motion along the singular direction is achieved by controlling the rate of $s_i(\mathbf{q})$. With the two possible assignments of the sign for the desired rate of $s_i(\mathbf{q})$, it is possible to select the posture of the manipulator among the two configurations that it can generally take when moving out of a singularity. The rate of $s_i(\mathbf{q})$ should be selected according to the desired velocity at the configuration when $|s_i(\mathbf{q})| = \eta_i$ from Equation 2, in order to achieve a smooth transition when crossing the boundary of the neighborhood of singularity.

4.3 Type 2 Singularity

At this type of singularities, the end-effector can only move in or about a direction that is orthogonal to the singular direction. To allow the end-effector to perform a motion in or about a given direction, first, the singular direction must be changed to be orthogonal to this direction. Controlling the direction of singularity can be accomplished by motions in the null space.

Let $\zeta_i(\mathbf{q})$ be the i$^{\text{th}}$ singular direction. The vector $\zeta_i(\mathbf{q})$ can be controlled by a potential function whose minimum corresponds to the desired singular direction, ζ_{id}. The control in the null space can be implemented following Equation 5.

Our control strategy for dealing with this type of singularity is to maintain a constant singular direction during motions in the neighborhood of singularity. This will prevent the large internal joint motions that are generally associated with end-effector motions in the neighborhood of the singularity.

For tasks involving trajectory following, the end-effector desired motion in the neighborhood of singularities should be designed to be in the subspace orthogonal to the singular direction.

5 Experimental Result

The above strategy has been implemented to control a PUMA 560 for the three different singularities shown in Figure 3. For some configurations, these singularities can occur simultaneously and the rank of the Jacobian can vary from 3 to 6. The minimum rank of the Jacobian corresponds to the configuration at which the end-effector reaches the highest point directly above the base.

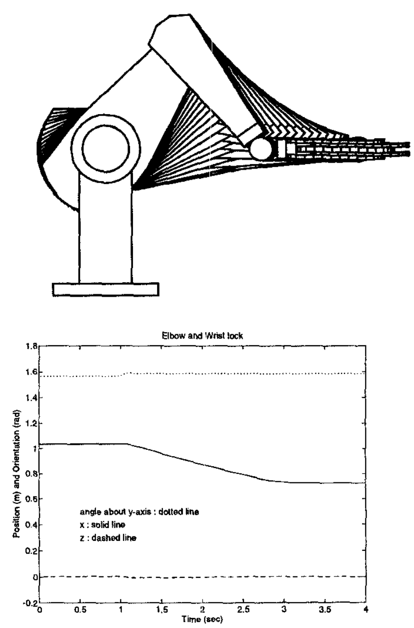

Figure 4: Elbow Lock and Wrist Lock: Experimental Response

In Figure 4, the end-effector is simultaneously moving out of two singularities: the elbow lock (Type 1) and wrist lock (Type 2). The goal is to translate along the singular direction, while maintaining all other positions and orientations. The resulting motion is shown in Figure 4.

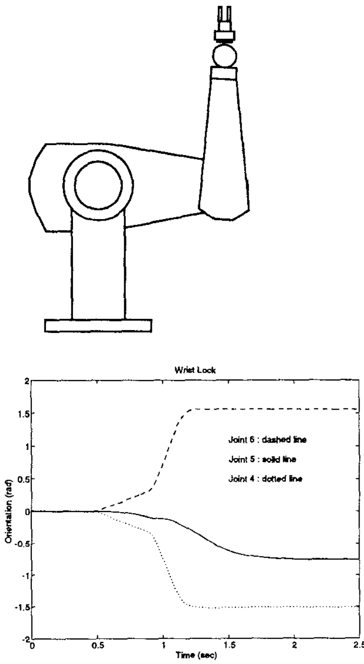

Figure 5: Wrist Lock: Experimental Response

In Figure 5, the end-effector is shown at a singular configuration of wrist lock (Type 2). The goal for the end-effector is to perform a rotation of 45° about the singular direction. This task is accomplished by first rotating the singular direction to a direction where it is orthogonal to its initial configuration. This is accomplished by the control discussed above, which involves internal joint motions in the null space. The resulting motions of joints 4, 5, and 6 are shown in Figure 5.

6 Conclusion

In this paper, we have proposed a classification of singularities into two types following the control characteristics of their null spaces: singularities at which the end-effector can be controlled in the singular directions using motion in the null space (Type 1); and singularities at which motion in the null space only affects the singular direction (Type 2). A general control strategy for these two types of singularities has been developed. The implementation of this strategy relies on the *dynamically consistent* force/torque relationship, which guarantees decoupled behavior between end-effector control and null space control of redundant mechanisms. The experimental results with a PUMA 560 illustrate the effectiveness of this strategy.

Acknowledgments

The financial support of Boeing, General Motors, Hitachi Construction Machinery, and NSF (grant IRI-9320017) is gratefully acknowledged. Many thanks to Diego Ruspini and David Williams for their comments and help during the preparation of the manuscript.

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