The Virtual Linkage: A Model for Internal Forces in Multi-Grasp Manipulation

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Abstract

We propose a model to characterize internal forces and moments during multi-grasp manipulation. The proposed approach is based on construction of a physical model, called the virtual linkage, *which is a closed chain mechanism that represents the object being manipulated. Forces and moments applied at the grasp points of this linkage cause joint forces and torques at its actuators. When these actuators are subjected to the opposing forces and torques, the virtual linkage becomes a locked structure. Internal forces in the object are then characterized by these forces and torques. This approach was used in a multi-arm robot system to manipulate objects while performing accurate control of internal forces.*

1 Introduction

There has been a great deal of interest in the control of internal forces for both multi-finger [3, 5, 7] and multi-arm [8, 9] robotic systems. For multiple arm coordination, internal force control is often used to compensate for kinematic errors and dynamic interaction forces. In grasping, internal forces are controlled in such a way as to satisfy friction constraints and prevent slip.

Whatever the motivation for internal force control, one must somehow characterize the internal forces associated with a given grasp configuration. To date, research dealing with internal forces and moments has focused essentially on two tasks; minimizing internal forces during manipulation [6], or determining a set of grasp forces that satisfy frictional and stability constraints for point contact without applying excessive force to the manipulated object [1, 2]. Typically these approaches establish the relationship between the forces applied to an object and the resultant of these forces at some reference point. They then proceed to solve this relationship using a generalized inverse. The basis for the nullspace associated with this generalized inverse is used to characterize the internal forces.

For instance, in [5], the *interaction force* between any two fingers in point contact with the manipulated object was defined to be the difference of the contact forces projected along the line joining the two contact points. The authors showed that the Moore-Penrose generalized inverse generates finger forces which have no component in this "interaction" force subspace.

While such methods allow closed-loop minimization of internal forces, they are not as useful for maintaining a specific set of non-zero internal forces. This is because their characterizations of the internal forces are not based on any physical model. The goal of this paper is to propose such a model; one which provides a realistic characterization of these forces.

2 The Virtual Linkage Model

To model the internal forces and moments acting on an object, it is important to define a model that gives a reasonable approximation of the actual object. When an object is being manipulated by a set of robot arms, its internal stress state is a complicated function of both its geometry and its motion. To simplify the analysis of these stresses, we consider a quasi-static case where object velocities and accelerations do not affect internal forces. Furthermore, we must approximate the continuous stress field with a discrete set of internal forces which can be independently controlled.

To characterize these internal forces, we introduce the concept of a *virtual linkage*.

A virtual linkage associated with an n-grasp manipulation task is a $6(n-1)$*-degree-of-freedom mechanism whose actuated joints characterize the object's internal forces and moments.*

Forces and moments applied at the grasp points of a virtual linkage cause joint forces and torques at its actuators. When these actuated joints are subjected to the opposing forces and torques, the virtual linkage becomes a statically determinate locked structure. The internal forces and moments in the object are then characterized by these forces and torques.

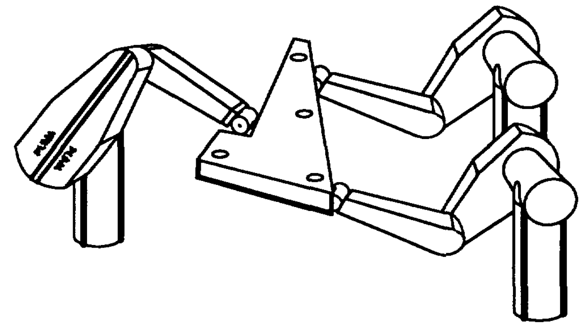

Figure 1: A multi-arm manipulation task

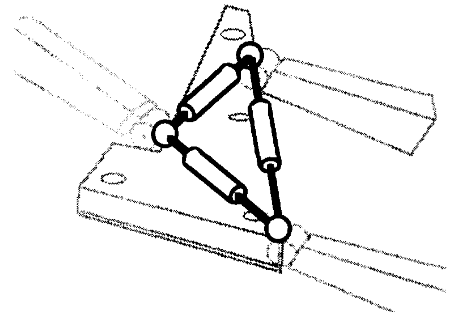

Figure 2: A Virtual Linkage Corresponding to the Grasp of Figure 1.

Consider a rigid body manipulation task being performed by three manipulators, as illustrated in Figure 1. If we assume that each manipulator grasps the object rigidly, and that none is in a singular configuration, the resulting system has eighteen actuator degrees of freedom. Since specifying the resultant forces and moments requires six degrees of freedom, twelve degrees of actuator redundancy exist. These twelve degrees of freedom can be used to produce twelve independent internal forces and moments.

The kinematic structure of the virtual linkage we propose for this manipulation task is one in which three actuated prismatic joints are connected by passive revolute joints to form a closed-chain, three-degree-of-freedom mechanism. A spherical joint with three actuators is then connected at each grasp point to resist internal moments.

The selection of this kinematic structure is motivated as follows. Since forces applied by the arms act to produce stresses throughout the object, they are best represented as a set of interactions between arms. This is the justification for actuated members in the virtual linkage. Applied moments, however, are local phenomena, and are best modeled by introducing actuated spherical joints at each grasp point.

The virtual linkage described above constitutes the basic structure for a virtual linkage with any number of grasps. To model additional grasp points, we first note that each new grasp introduces three internal forces and three internal moments. These are accounted for by introducing three more prismatic actuators and one more spherical joint to the linkage, as illustrated in the example below.

The virtual linkage corresponding to the manipulation task in Figure 1 is shown in Figure 2. This virtual linkage has three linearly actuated "members", along which we can specify three independent internal forces. In general, any virtual linkage associated with an n-grasp manipulation task requires $3(n-2)$ actuated members. We can independently specify forces in these $3(n-2)$ members, along with n internal moments at the grasp points (corresponding to $3n$ actuator degrees of freedom).

2.1 The Grasp Description Matrix

Consider again the virtual linkage shown in Figure 2. Forces and moments from the individual arms are applied at the joints of this linkage. Our goal is to relate these forces and moments to the resultant and internal forces acting on the object.

To begin, we assume that only forces act at the grasp points; this assumption will be relaxed at a later stage in the analysis. Let e_{ij} be the unit vector along the link from grasp i to grasp j, and let r_i be the vector from the reference point O to grasp point i as shown in Figure 3. Let $\mathbf{f}_i$ be the force applied at grasp point i, and let $\mathbf{f}$ be the vector

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \end{bmatrix}.$$

If the virtual linkage was at static equilibrium, the relationship between the internal forces $\mathbf{t} = [t_1 \; t_2 \; t_3]^T$ and the applied forces, $\mathbf{f}$, would be given by

$$\mathbf{f} = E\mathbf{t}; \tag{1}$$

where

$$E = \begin{bmatrix} -\mathbf{e}_{12} & 0 & \mathbf{e}_{31} \\ \mathbf{e}_{12} & -\mathbf{e}_{23} & 0 \\ 0 & \mathbf{e}_{23} & -\mathbf{e}_{31} \end{bmatrix}. \tag{2}$$

When the linkage is not at static equilibrium, we decompose the forces applied at the grasp points into two parts; those which cause tensions, and those which

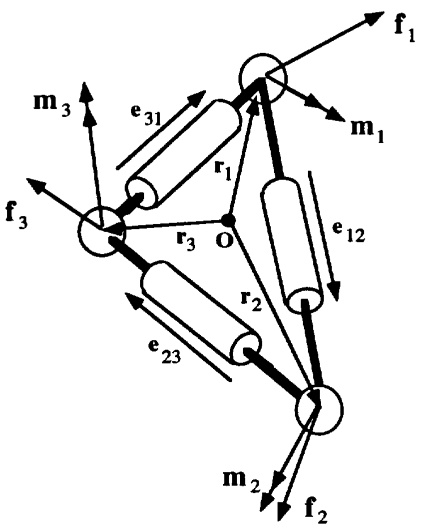

Figure 3: Virtual Linkage Nomenclature

produce the desired resultant without affecting the tensions in the virtual linkage:

$$\mathbf{f} = E\mathbf{t} + \mathbf{f}_e. \tag{3}$$

The solution to this set of equations is unique, and is given by

$$\mathbf{t} = \overline{E}(\mathbf{f} - \mathbf{f}_e); \tag{4}$$

where $\overline{E}$ is a left inverse,

$$\overline{E} = (E^T E)^{-1} E^T. \tag{5}$$

Furthermore, $\mathbf{f}_e$ produces no tensions by definition, and $\overline{E}\mathbf{f}_e = 0$. Thus, the internal forces are given by

$$\mathbf{t} = \overline{E}\mathbf{f}. \tag{6}$$

The relationships between the resultant force, $\mathbf{f}_r$, the resultant moment $\mathbf{m}_r$, and the applied forces $\mathbf{f}$ are given by:

$$\begin{bmatrix} \mathbf{f}_r \\ \mathbf{m}_r \end{bmatrix} = W_f \mathbf{f}; \tag{7}$$

where

$$W_f = \begin{bmatrix} I_3 & I_3 & I_3 \\ \hat{\mathbf{r}}_1 & \hat{\mathbf{r}}_2 & \hat{\mathbf{r}}_3 \end{bmatrix}; \tag{8}$$

In this equation, $\hat{\mathbf{r}}_i$ is the cross product operator corresponding to the vector $\mathbf{r}_i$ shown in Figure 3, and I_3 is the 3×3 identity matrix.

The relationship between applied forces, resulting forces and internal forces is

$$\begin{bmatrix} \mathbf{f}_r \\ \mathbf{m}_r \\ \mathbf{t} \end{bmatrix} = G_f \mathbf{f} \tag{9}$$

where

$$G_f = \begin{bmatrix} W_f \\ \hline \overline{E} \end{bmatrix}; \tag{10}$$

G_f is a 9×9 matrix, which is full-rank when the three grasp points are not collinear. It is called the grasp description matrix, which relates the forces applied at each grasp to the resultant and internal forces in the object. Its subscript signifies that this grasp description matrix is for the special case in which the applied moments are zero.

The inverse of the matrix G_f provides the forces required at the grasp points to produce a desired resultant and specified internal forces. This matrix is

$$G_f^{-1} = \begin{bmatrix} \overline{W}_f & E \end{bmatrix}; \tag{11}$$

where

$$\overline{W}_f = W_f^T (W_f W_f^T)^{-1}. \tag{12}$$

We now have an explicit formula for the grasp description matrix and its inverse for the case of point contact which accurately reflect tensions in the virtual linkage.

Note that the forces $\mathbf{f}_e$ in equation 3 correspond to

$$\mathbf{f}_e = \overline{W}_f \begin{bmatrix} \mathbf{f}_r \\ \mathbf{m}_r \end{bmatrix}.$$

Now we extend the analysis of internal forces to the case in which moments are also applied at the grasp points as shown in Figure 3. These applied moments will be represented by the vector $\mathbf{m}$:

$$\mathbf{m} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}.$$

The resultant force and moment acting on the system are

$$\begin{bmatrix} \mathbf{f}_r \\ \mathbf{m}_r \end{bmatrix} = W \begin{bmatrix} \mathbf{f} \\ \mathbf{m} \end{bmatrix}; \tag{13}$$

with

$$W = [\ W_f \quad W_m\ ]; \tag{14}$$

where

$$W_m = \begin{bmatrix} 0_3 & 0_3 & 0_3 \\ I_3 & I_3 & I_3 \end{bmatrix}.$$

The application of moments results in an additional nine degrees of actuator redundancy which correspond to the nine internal moments. Given their local nature, internal moments have been modeled as spherical joints at each grasp point in the virtual linkage model. The internal moment at grasp i, denoted τ_i, will therefore be defined to be equal to the moment applied at that grasp point.

The resolution of internal forces is affected by the internal moments. However, this effect will be neglected in the following derivation. As a consequence, when internal moments exist, the internal forces will not exactly equal the forces required in the virtual actuators to lock the mechanism. But, since the virtual linkage is already an approximation to the actual object, the values obtained for internal forces will still provide a complete characterization of the internal forces in the object. In addition, internal moments generally result in large localized stresses at the grasp point and should be minimized for delicate manipulation tasks. In this case the computed value of the internal forces will be exact.

With the above assumption, the internal forces are still given by equation 6. The relationship expressing the resultant and internal forces in terms of the applied forces and moments is

$$\begin{bmatrix} \mathbf{f}_r \\ \mathbf{m}_r \\ \mathbf{t} \\ \tau \end{bmatrix} = G \begin{bmatrix} \mathbf{f} \\ \mathbf{m} \end{bmatrix}; \tag{15}$$

where G is the 18×18 *grasp description matrix* given by

$$G = \begin{bmatrix} W_f & W_m \\ E & 0_{3\times 9} \\ 0_9 & I_9 \end{bmatrix}. \tag{16}$$

Its inverse is

$$G^{-1} = \begin{bmatrix} \overline{W}_f & E & \overline{W}_f W_m \\ 0_{9\times 6} & 0_{9\times 3} & I_9 \end{bmatrix}. \tag{17}$$

The virtual linkage model can be extended to any number of grasp points. Each additional grasp point results in six new actuator degrees of freedom which must be characterized by the virtual linkage. This will be accomplished by connecting new grasp points to existing grasp points through three actuated virtual members and one spherical joint. For instance, in the example presented above, a fourth grasp is added as shown in Figure 4.

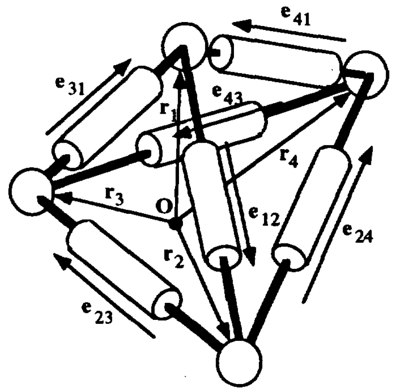

Figure 4: A Four Grasp Virtual Linkage

For some grasp configurations, the rank of the grasp description matrix G might be deficient. In such a situation, the virtual linkage is at a kinematic singularity; two or more adjacent members are aligned. This implies that the system has lost the ability to independently specify all internal forces and moments.

2.2 A Two-Grasp Analysis

The virtual linkage model can also be used to describe two-arm manipulation. In this case, however, the arms are unable to apply a moment along the line joining the two grasp points using forces alone. Because of this, there is only one component of internal moment in this direction. This internal moment will be defined by the difference between the two applied moments in that direction. Internal moments in the orthogonal directions are defined as described above.

Let us consider the case of two arms manipulating a beam as shown in Figure 5. To simplify the relationship between applied and internal forces, we represent the applied forces and moments in a coordinate system that is fixed in the object frame and whose x-axis is aligned with the virtual member. We can then write the relationship between applied forces and the single internal force, t as

$$\begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix} = Et; \tag{18}$$

where

$$E = [\ -1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0\ ]^T.$$

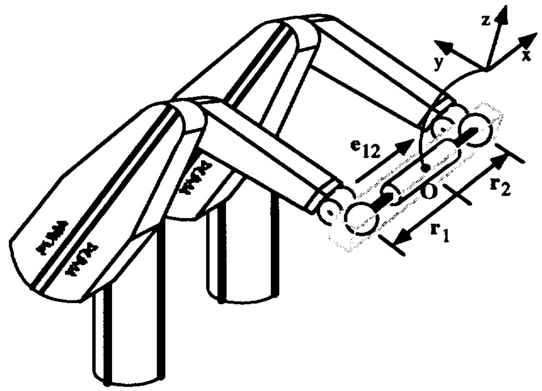

Figure 5: A Two-Grasp Manipulation Task

Solving for t gives

$$t = \overline{E}\begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix}; \qquad (19)$$

where

$$\overline{E} = \frac{1}{2}[\ -1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0\ ].$$

The resultant force on the object is described by

$$\begin{bmatrix} \mathbf{f}_r \\ \mathbf{m}_r \end{bmatrix} = [\ W_f \quad W_m\ ]\begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix}; \qquad (20)$$

with

$$W_f = \begin{bmatrix} I_3 & I_3 \\ \hat{\mathbf{r}}_1 & \hat{\mathbf{r}}_2 \end{bmatrix}; W_m = \begin{bmatrix} 0_3 & 0_3 \\ I_3 & I_3 \end{bmatrix};$$

where $\hat{\mathbf{r}}_i$ is the cross product operator for the vector $\mathbf{r}_i$ from the control reference point O to grasp i.

Finally, we can write the relationship between applied and internal moments as

$$\tau = \tilde{I}\begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix}; \qquad (21)$$

where

$$\tilde{I} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that there are only five internal moments instead of the six that one might expect. This is because the sum of moments along the virtual member contributes directly to the resultant. Their difference in this direction is defined as an internal moment. Equations 19, 20, and 21 give the grasp description matrix for the two-arm grasp. If the reference point, O, is centered between the grasp points, G is given explicitly by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{r}{2} & 0 & 0 & -\frac{r}{2} & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -\frac{r}{2} & 0 & 0 & \frac{r}{2} & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \qquad (22)$$

where r is the distance between grasps. Different reference points can be used by performing a simple transformation. Note that the inverse of G is no longer given by equation 17 since G is not in the form of equation 16. Using row and column operations to transform G into a block upper-trianglular matrix with full rank blocks on the diagonal, inverting, and undoing these operations yields the following analytical expression for G^{-1}:

$$G^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{r} & 0 & 0 & 0 & \frac{1}{r} & 0 & \frac{1}{r} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{r} & 0 & 0 & 0 & -\frac{1}{r} & 0 & -\frac{1}{r} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{r} & 0 & 0 & 0 & -\frac{1}{r} & 0 & -\frac{1}{r} \\ 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{r} & 0 & 0 & 0 & \frac{1}{r} & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (23)$$

3 Experimental Results

The virtual linkage model has been implemented using COSMOS, an object-level control system based on the operational space approach. It has been coupled with the augmented object model [4] which describes the control of multiple arms. This system was

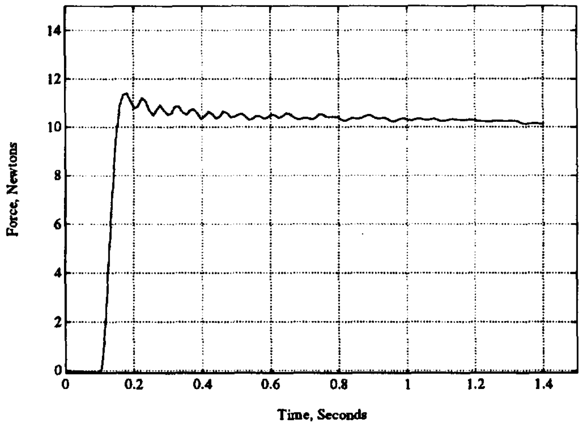

Figure 6: Two-Arm Response to a 10 N Step Input

then used to implement closed-loop control of internal forces and moments for two PUMA 560 manipulators.

Figure 6 shows the internal force response for two arms holding a piece of plexiglass. Here, the system is asked to apply a 10 Newton tension.

4 Conclusions

A model has been proposed for the description of internal forces and moments in multi-grasp manipulation. This model is based on the construction of a physical model, called the ***virtual linkage***, a closed chain mechanism representing the object being manipulated. Forces and moments applied at the grasp points of this linkage cause joint forces and torques at its actuators. Internal forces in the object are then characterized by these actuator forces.

Explicit formulas have been derived to express these internal forces in terms of the applied forces and moments at the grasp points for any number of grasps. These formulas are simple enough to be implemented in real-time. This approach has been implemented in COSMOS, an object-level control system, to perform manipulation tasks while controlling internal forces.

Acknowledgments

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