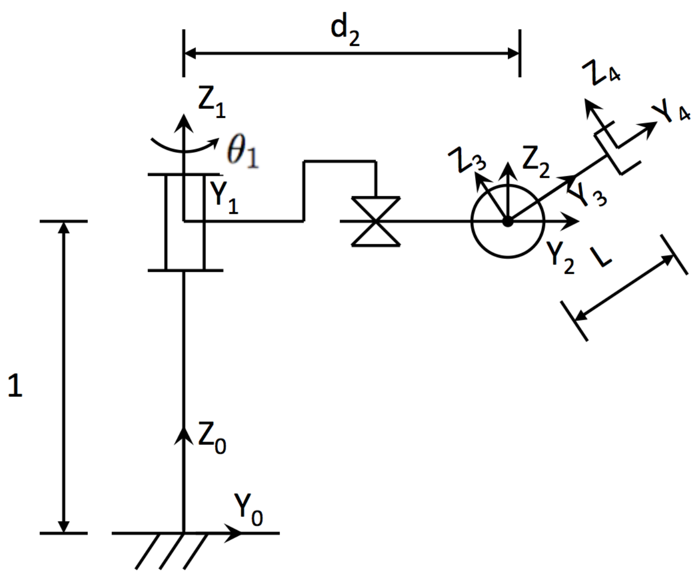
CS225a Autumn 2014 Homework #0 Solution

Tutorial #0 Q1

For both problem 1 and 2, the following RPR robot is used. Notice that the frames are not assigned according to DH conventions. The frames are assigned such that the transformation at joint angle = 0 is translation only. This would generate more zero entries and identify matrices and simplify the computation.



(a) The transformation matrices from link frame to parent frame are as follow.

$$ {}^1_0T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & -s_3 & 0 \\ 0 & s_3 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Given the end effector vector expressed in frame 3

$$ {}^3\vec{p} = \begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix} $$

we can compute its position vector expressed in the root frame from the homogenous transformation matrix.

$$ {}^0\vec{p} = ({}^1_0T)({}^2_1T)({}^3_2T)\,{}^3\vec{p} = \begin{bmatrix} -\,s_1(d_2 + c_3L) \\ c_1(d_2 + c_3L) \\ s_3L + 1 \end{bmatrix} $$

(b) Assuming L = 1, we then select three sets of generalized coordinate values to perform the test.

$$ q_1 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, q_2 = \begin{bmatrix} 0 \\ 2 \\ \frac{\pi}{2} \end{bmatrix}, q_3 = \begin{bmatrix} \frac{\pi}{2} \\ 2 \\ \frac{\pi}{2} \end{bmatrix} $$

Based on the homogeneous transformation matrix, we get the following end effector positions.

$$ {}^0\vec{p_1} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, {}^0\vec{p_2} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, {}^0\vec{p_3} = \begin{bmatrix} -\,2 \\ 0 \\ 2 \end{bmatrix} $$

Based on the modified tutorial code, we get the following end effector positions.

$$ {}^0\vec{p_1} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, {}^0\vec{p_2} = \begin{bmatrix} 0 \\ 2.0008 \\ 2 \end{bmatrix}, {}^0\vec{p_3} = \begin{bmatrix} -\,2.0008 \\ 0.0016 \\ 2 \end{bmatrix} $$

The results generated from the code match those calculated by hand.

Tutorial #0 Q2

(a) If the end-effectors of two arms are connected, then one end-effector will lose 6-DOF in general. However, one end-effector may have less than 6-DOF to begin with, or both might have different DOFs.

Given:
Robot 1, N DOFs at end-effector (max 6)
Robot 2, M DOFs at end-effector (max 6)

DOF lost is:
M+N-DOF(intersection(M-DOF,N-DOF)

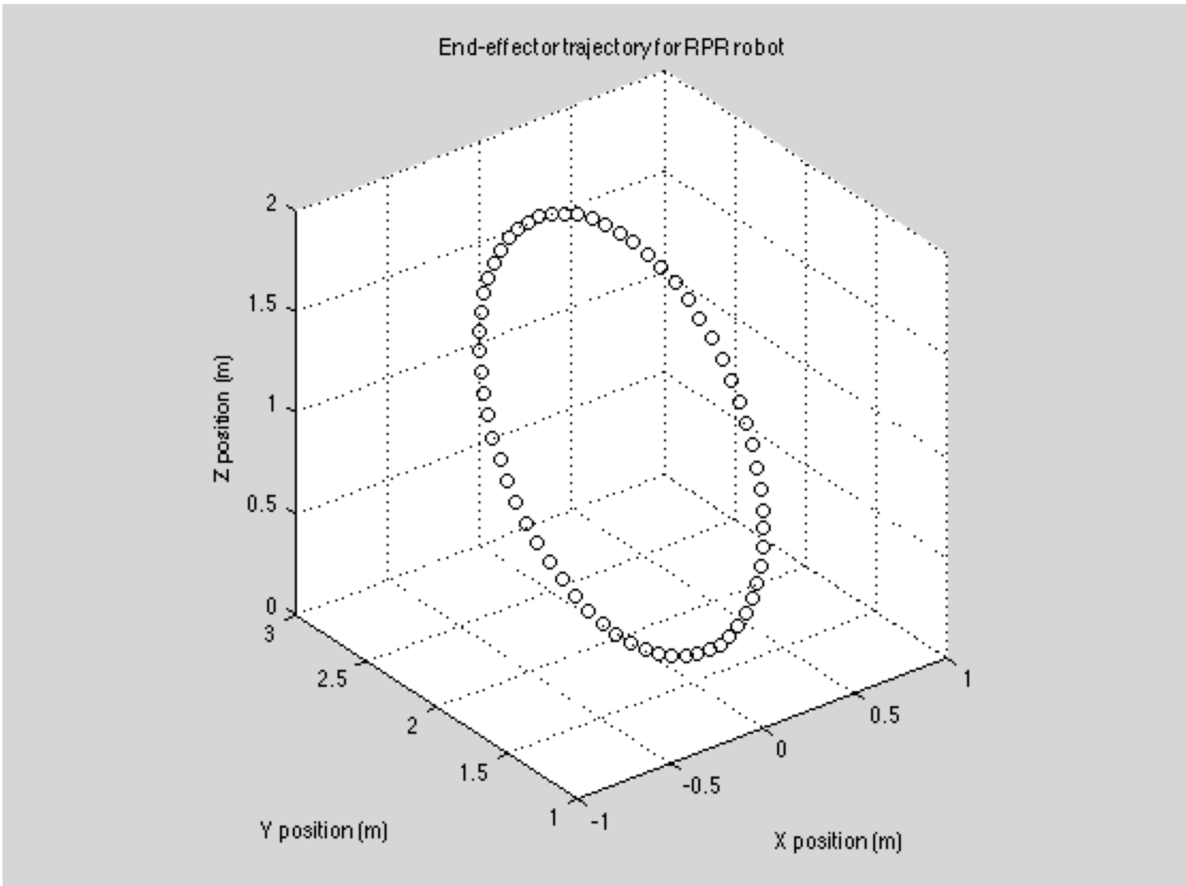
For example:
Robot 1, 5-DOF at end effector $(x, y, z, \theta_1, \theta_2)$
Robot 2, 5-DOF at end effector $(x, y, \theta_1, \theta_2, \theta_3)$
intersection(M-DOF,N-DOF) = $(x, y, \theta_1, \theta_2)$
DOF(intersection) = 4
M+N-DOF(interesection) = 5 + 5 - 4 = 6.

6 DOFs have been lost:
Robot 1 lost $(x, y, z, \theta_1, \theta_2)$, which robot 2 now "controls".
Robot 2 lost (θ_3), which is constrained by robot 1's structure.

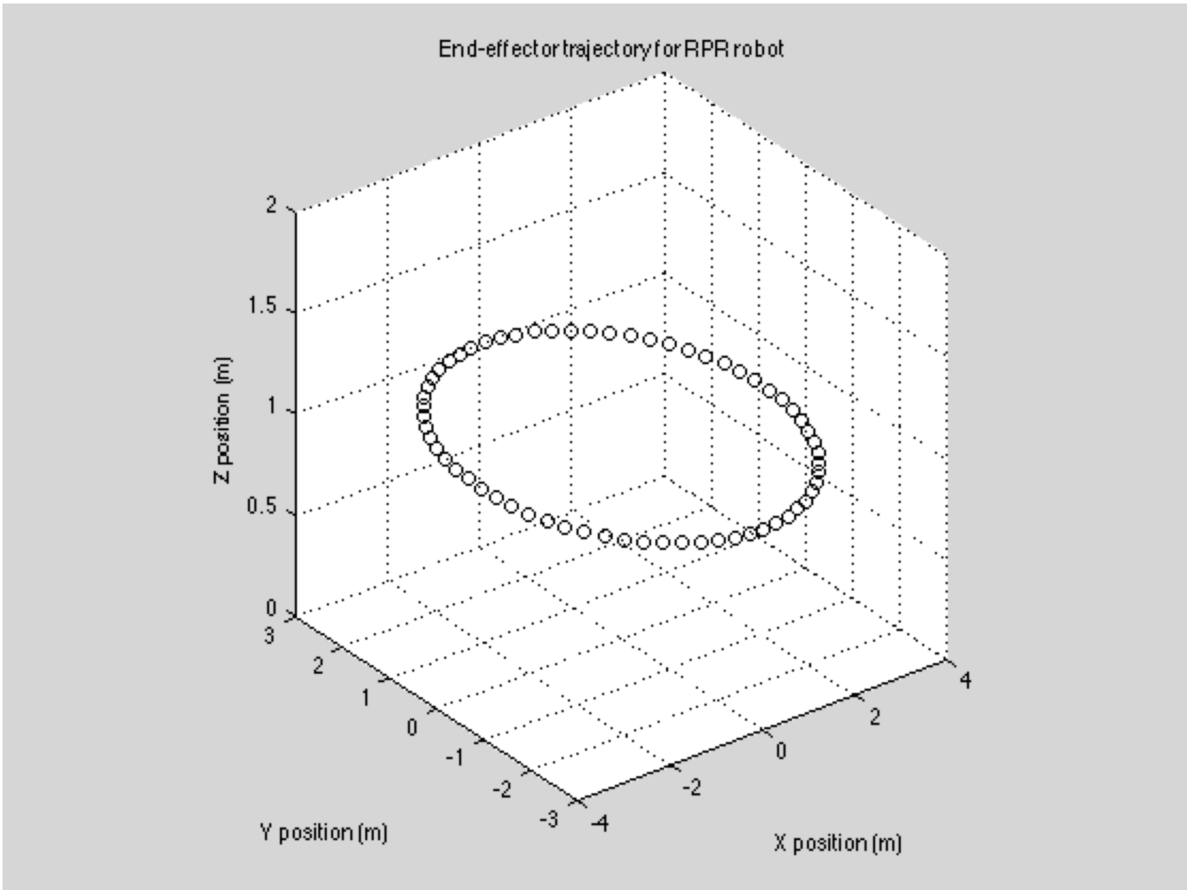
The same goes for any rigid body loop connection, not just end-effectors.

Tutorial #1 Q1

(a) With the first revolute and the prismatic joint set to 0 and 2, respectively, and the third revolute joint moving between -π and π, the end-effector position is shown in the following graph.



(b) With the third revolute and the prismatic joint set to 0 and 2, respectively, and the first revolute joint moving between -π and π, the end-effector position is shown in the following graph.



Tutorial #1 Q2

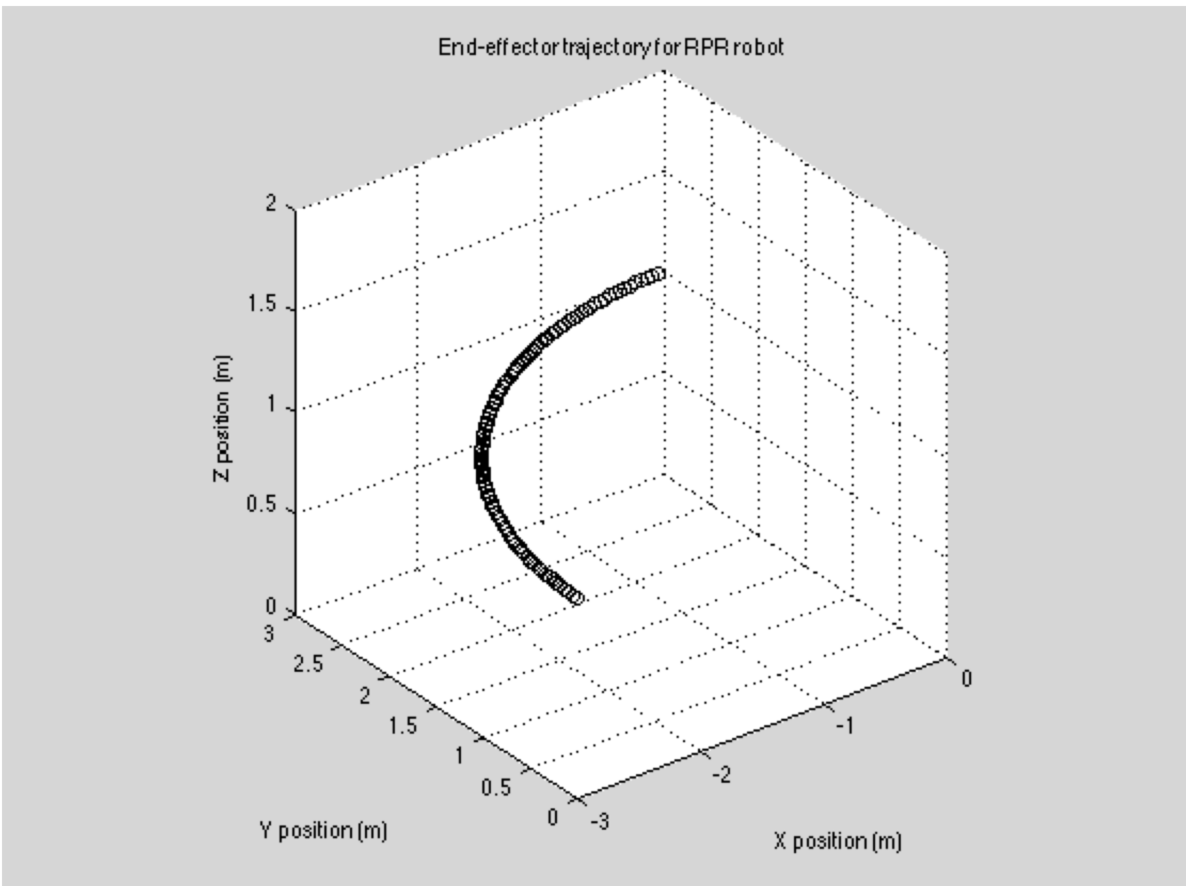
(a) We choose two reachable points on the x-y plane (z = 1).

$$ {}^{0}\vec{p_a} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, {}^{0}\vec{p_b} = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} $$

Then the corresponding two sets of generalized coordinates (joint angle / length) are

$$ q_a = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, q_b = \begin{bmatrix} \frac{\pi}{2} \\ 2 \\ 0 \end{bmatrix} $$

(b) After linearly interpolate q between q_a and q_b, the resulting end-effector positions ares shown in the following plot.



Obviously, the end-effector is NOT moving in a straight line. Notice that the joint space cannot be linearly transformed into the task space in general.