Trajectory Tracking

Trajectory: x_d, $\dot{x}_d$, $\ddot{x}_d$

$$F^* = I_{m_0}\ddot{x}_d - k_v(\dot{x} - \dot{x}_d) - k_p(x - x_d)$$

$$(\ddot{x} - \ddot{x}_d) + k_v(\dot{x} - \dot{x}_d) + k_p(x - x_d)$$

or $\ddot{\varepsilon}_X + k_v\dot{\varepsilon}_X + k_p\varepsilon_X = 0$

with $\varepsilon_X = x - x_d$

In joint space

$$\ddot{\varepsilon}_q + k_v\dot{\varepsilon}_q + k_p\varepsilon_q = 0$$

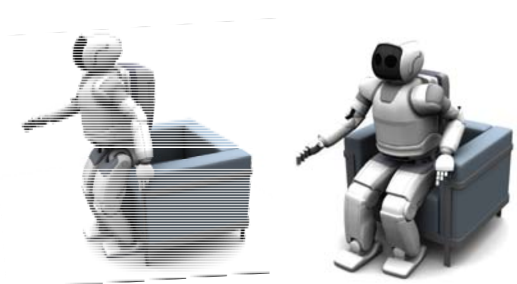
with $\varepsilon_q = q - q_d$

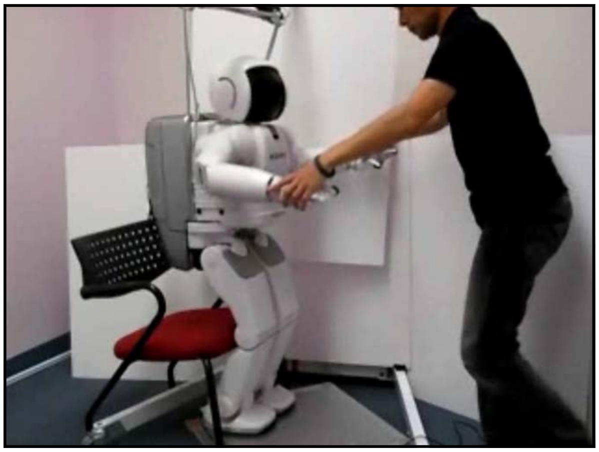
Compliant Motion Control

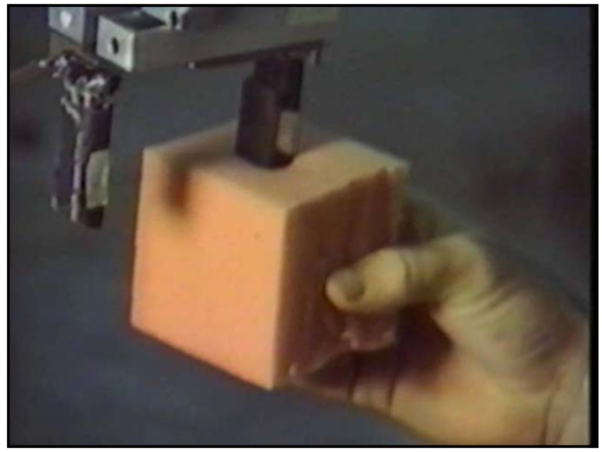
Advanced Manipulation Capabilities

Compliant Manipulation Primitives

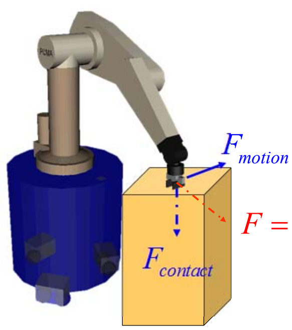
Multi-contact Manipulation





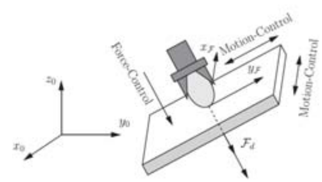


Unified Motion/Force Control



$F = F_{motion} + F_{contact}$

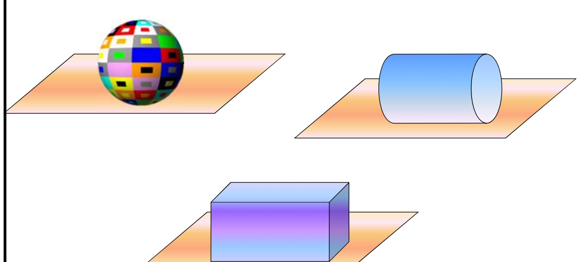
Unified Motion/Force Control



- Generalized Selection Matrix
- Dynamic Model (Homogeneity)

$$\Lambda_0(x)\dot{\vartheta} + \mu_o(x,\vartheta) + p_0(x) + F_{contact} = F_0$$

Task Description

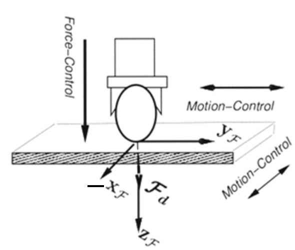


Task Specification

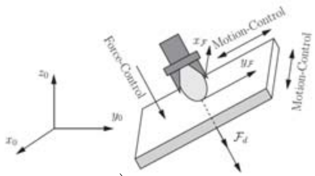
$$F = \Sigma F_{motion} + \bar{\Sigma} F_{force}$$

Selection matrix

$$\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \ \bar{\Sigma} = I - \Sigma$$



Generalized Selection Matrix



R_f $\quad \Sigma \rightarrow \Omega$

$f_{(R_f)} = R_f f_{(R_0)}$

Selection in R_f

$\bar{\Sigma} f_{R_f}$

Selection in R_0

$\underbrace{R_f^T \bar{\Sigma} R_f}_{\bar{\Omega}} f_{R_0} \Leftarrow R_f^T \bar{\Sigma} f_{R_f}$

Generalized Selection Matrix

$$\Sigma_F = \begin{pmatrix} \sigma_{F_x} & 0 & 0 \\ 0 & \sigma_{F_y} & 0 \\ 0 & 0 & \sigma_{F_z} \end{pmatrix}; \quad \bar{\Sigma}_F = I_3 - \Sigma_F$$

$$\Sigma_M = \begin{pmatrix} \sigma_{M_x} & 0 & 0 \\ 0 & \sigma_{M_y} & 0 \\ 0 & 0 & \sigma_{M_z} \end{pmatrix}; \quad \bar{\Sigma}_M = I_3 - \Sigma_M$$

Generalized Selection Matrix

$$\Omega = \begin{pmatrix} R_F^T \, \Sigma_F \, R_F & 0 \\ 0 & R_M^T \, \Sigma_M \, R_M \end{pmatrix}$$

$$\bar{\Omega} = \begin{pmatrix} R_F^T \, \bar{\Sigma}_F \, R_F & 0 \\ 0 & R_M^T \, \bar{\Sigma}_M \, R_M \end{pmatrix}$$

Basic Dynamic Model

Operational force $\stackrel{?}{\equiv}$ Forces & Moments

$$\dot{x} = J(q)\dot{q}$$

$$\Gamma = J^T(q) F$$

Linear & Angular Velocities $\qquad$ Forces & Moments

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = J_0(q)\dot{q}$$

$$\vartheta \triangleq \begin{pmatrix} v \\ \omega \end{pmatrix} \qquad F_0 \triangleq \begin{pmatrix} f \\ m \end{pmatrix}$$

$$\dot{x} = J\,\dot{q} = E(x) J_0\,\dot{q} \qquad \tau = J^T F = J_0^T (E^T F)$$

$$\dot{x} = E \begin{pmatrix} v \\ \omega \end{pmatrix} \qquad \begin{pmatrix} f \\ m \end{pmatrix} = E^T F$$

Basic Dynamic Model

$$\Lambda_0 = E^T \Lambda E$$

$$\Lambda \ddot{x} + \mu(x, \dot{x}) + p(x) = F$$

$$\Downarrow E^T$$

$$\boxed{\Lambda_0 \dot{\vartheta} + \mu_0(x, \vartheta) + p_0(x) = F_0}$$

with $\quad \vartheta \triangleq \begin{pmatrix} v \\ \omega \end{pmatrix}$

Orientation Representation

x_r

x_{rd}

$$\delta x_r = x_r - x_{rd}$$

$$\dot{x}_r = E_r\,\omega$$

Instantaneous Angular Error

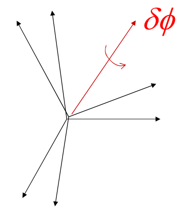
$$\delta x_r = E_r\,\delta\phi$$

Instantaneous Angular Error

$$\dot{x}_r = E_r\omega$$

$$\delta x_r = E_r\,\delta\phi$$

$$\boxed{\delta\phi = E_r^{+}\,\delta x_r}$$



Control - Position Errors

$$\left(x - x_d\right) = \begin{pmatrix} x_p - x_{pd} \\ x_r - x_{rd} \end{pmatrix}$$

$$\dot{x}_r = E_r\,\omega$$

$$\underline{\delta x_r} = E_r\,\underline{\delta\phi} \Rightarrow \begin{pmatrix} x_p - x_{pd} \\ \delta\phi \end{pmatrix} \text{ Error Vector}$$

$$\delta x_r = \left(x_r - x_{r(d)}\right) = E_r\,\delta\phi$$

$$\boxed{\delta\phi = E_r^{+}\left(x_r - x_{r(d)}\right)}$$

Goal Position

$$x_d = \begin{bmatrix} x_{pd} \\ x_{rd} \end{bmatrix}$$

$$f^{*} = -k_p\left(x_p - x_{pd}\right) - k_v\dot{x}_p$$

$$m^{*} = -k_p\delta\phi - k_v\omega$$

with $\delta\phi = E_r^{+}\left(x_r\right)\left(x_r - x_{rd}\right)$

Closed loop

$$I.\ddot{x}_p + k_v\dot{x}_p + k_p\left(x_p - x_{pd}\right) = 0$$

$$I\,\dot{\omega} + k_v\omega + k_p\delta\phi = 0$$

Direction Cosines

$$x_r = \left(r_1^T r_2^T r_3^T\right)^T$$

$$x_{rd} = \left(r_{1d}^T\, r_{2d}^T\, r_{3d}^T\right)^T$$

$$E_r^{+} = \frac{1}{2}E_r^T$$

The angular rotation error

$$\delta\phi = -\frac{1}{2}\left(\hat{r}_1 r_{1d} + \hat{r}_2 r_{2d} + \hat{r}_3 r_{3d}\right)$$

Euler Parameters

The end-effector orientation

$$x_r = \lambda = \left(\lambda_0\lambda_1\lambda_2\lambda_3\right)^T$$

The desired orientation

$$\lambda_d = \left(\lambda_{0d}\lambda_{1d}\lambda_{2d}\lambda_{3d}\right)^T$$

The angular rotation error

$$\boxed{\delta\phi = E_R^{+}(\lambda)\lambda_d}$$

$$E_R^{+}(\lambda) = 2\begin{pmatrix} -\lambda_1 & \lambda_0 & -\lambda_3 & \lambda_2 \\ -\lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1 \\ -\lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0 \end{pmatrix}$$

Motion Tracking $(x_{pd}\ ,\ \dot{x}_{pd}\ ,\ \ddot{x}_{pd})$

$$F^* = \ddot{x}_{pd} - k_p\left(x_p - x_{pd}\right) - k_v\left(\dot{x}_p - \dot{x}_{pd}\right)$$

Closed loop

$$I.\ddot{\varepsilon}_x + k_v\dot{\varepsilon}_x + k_p\varepsilon_x = 0$$

with

$$\varepsilon_{x_p} = x_p - x_{pd}$$

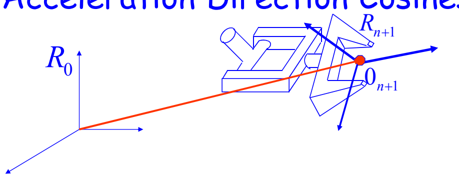
Angular Acceleration

$$\dot{x}_r = E_r\omega$$

$$\ddot{x}_r = E_r\dot{\omega} + \dot{E}_r\omega$$

$$\boxed{\dot{\omega} = E_r^+\ddot{x}_r - E_r^+\dot{E}_r\omega}$$

Acceleration Direction Cosines



The orientation is described by

$$x_r = \left(r_1^T\, r_2^T\, r_3^T\right)^T$$

$$r_1 = x_{(n+1)};\ r_2 = y_{(n+1)};\ r_3 = z_{(n+1)};$$

The second time derivatives

$$\frac{d^2\mathbf{x}_{(n+1)}}{dt^2} = -\mathbf{x}_{(n+1)} \times \dot{\omega} + \left(\mathbf{x}_{(n+1)} \times \omega\right) \times \omega$$

$$\frac{d^2\mathbf{y}_{(n+1)}}{dt^2} = -\mathbf{y}_{(n+1)} \times \dot{\omega} + \left(\mathbf{y}_{(n+1)} \times \omega\right) \times \omega$$

$$\frac{d^2\mathbf{z}_{(n+1)}}{dt^2} = -\mathbf{z}_{(n+1)} \times \dot{\omega} + \left(\mathbf{z}_{(n+1)} \times \omega\right) \times \omega$$

However

$$\mathbf{u} \times \mathbf{v} \times \mathbf{w} = \left(\mathbf{u}^T\mathbf{v}\right)\mathbf{w} - \left(\mathbf{v}^T\mathbf{w}\right)\mathbf{u}$$

This yields

$$\ddot{\mathbf{x}}_r = E(\mathbf{x}_r)\dot{\omega} + R(\mathbf{x}_r,\omega)\omega - (\omega^T\omega)\mathbf{x}_r$$

where

$$R(\mathbf{x}_r,\omega) = \begin{vmatrix} \left(r_1^T\omega\right)I_3 \\ \left(r_2^T\omega\right)I_3 \\ \left(r_3^T\omega\right)I_3 \end{vmatrix}$$

Acceleration Direction Cosines

$$\dot{\omega}_d = \frac{1}{2}E^T\,\ddot{x}_{rd} + \frac{1}{2}R^T(x_r,\omega)\dot{x}_{rd}$$

Euler Parameters

The acceleration associated with Euler parameters

$$\ddot{\lambda} = \frac{1}{4}\overset{\vee}{\lambda}\,\dot{\omega} - \frac{1}{2}(\omega^T\omega)\lambda$$

since $\overset{\vee}{\lambda}{}^T\,\lambda = 0$ $\quad \overset{\vee}{\lambda} = \begin{pmatrix} -\lambda_1 & -\lambda_2 & -\lambda_3 \\ \lambda_0 & \lambda_3 & -\lambda_2 \\ -\lambda_3 & \lambda_0 & \lambda_1 \\ \lambda_2 & -\lambda_1 & \lambda_0 \end{pmatrix}$

Euler Parameters

The angular acceleration vector

$$\dot{\omega} = 4\,\overset{\vee}{\lambda}{}^T\,\ddot{\lambda}$$

The desired angular acceleration

$$\dot{\omega}_d = 4\,\overset{\vee}{\lambda}_d{}^T\,\ddot{\lambda}_d$$

Motion Tracking $(x_{pd}\ ,\ \dot{x}_{pd}\ ,\ \ddot{x}_{pd})$

$$F^* = \ddot{x}_{pd} - k_p\left(x_p - x_{pd}\right) - k_v\left(\dot{x}_p - \dot{x}_{pd}\right)$$

Closed loop

$$I.\ddot{\varepsilon}_x + k_v\dot{\varepsilon}_x + k_p\varepsilon_x = 0$$

with

$$\varepsilon_{x_p} = x_p - x_{pd}$$

Motion Tracking $(x_{pd}\ ,\ \dot{x}_{pd}\ ,\ \ddot{x}_{pd})$

$$f^* = \ddot{x}_{pd} - k_p\left(x_p - x_{pd}\right) - k_v\left(\dot{x}_p - \dot{x}_{pd}\right)$$

$$m^* = \dot{\omega}_d - k_p\delta\phi - k_v\left(\omega - \omega_d\right)$$

with $\quad \delta\phi = E_r^+\left(x_r - x_{rd}\right)$

$$\omega_d = E_r^+\left(x_{rd}\right)\dot{x}_{rd}$$

and

$$\dot{\omega}_d = E_r^+\left(x_{rd}\right)\ddot{x}_{rd} - E_r^+\left(x_{rd}\right)\dot{E}\left(x_{rd}\right)\omega_d$$

Closed loop

$$\left(\ddot{x}_p - \ddot{x}_{pd}\right) + k_v\left(\dot{x}_p - \dot{x}_{pd}\right) + k_p\left(x_p - x_{pd}\right) = 0$$

$$\left(\dot{\omega} - \dot{\omega}_d\right) + k_v\left(\omega - \omega_d\right) + k_p\delta\phi = 0$$

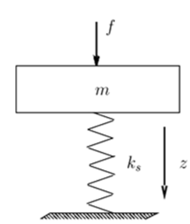
A Mass Spring System

System

$$m\ddot{z} + k_s z = f$$

$$f_s = k_s z$$

$$m\frac{1}{k_s}\ddot{f}_s + f_s = f$$



System $\quad m\frac{1}{k_s}\ddot{f}_s + f_s = f$

Control

$$f = f_s + m \bullet f_{comp}$$

$$f = f_s - m\left[k_f\left(f_s - f_d\right) + k_{v_f}\dot{f}_s\right]$$

Control-loop System

$$\ddot{f}_s + k_s k_{v_f}\dot{f}_s + k_s k_f\left(f_s - f_d\right) = 0$$

Static Equilibrium

$$f_s = f_d$$

End-Effector/Sensor System

$$\Lambda_0\dot{\vartheta} + \mu_0(x,\vartheta) + p_0(x) + F_{contact} = F_0$$

Unified Control

$$F_0 = F_{motion} + F_{force}$$

$$F_{motion} = \hat{\Lambda}_0\,\Omega\,F^*_{motion} + \hat{\mu}_0 + \hat{P}_0$$

$$F_{force} = \hat{\Lambda}_0\,\bar{\Omega}\,F^*_{force} + F_{sensor}$$

End-Effector/Sensor System

$$\Lambda_0\dot{\vartheta} + \mu_0(x,\vartheta) + p_0(x) + F_{contact} = F_0$$

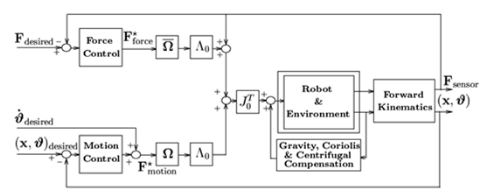
Unified Control

$$F_0 = F_{motion} + F_{force}$$

$$F_{motion} = \hat{\Lambda}_0\,\Omega\,F^*_{motion} + \hat{\mu}_0 + \hat{P}_0$$

$$F_{force} = \hat{\Lambda}_0\,\bar{\Omega}\,F^*_{force} + \bar{\Omega}\,F_{desired}$$

Unified Motion & Force Control

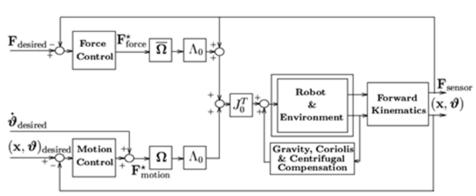


Two decoupled Subsystems

$$\Omega\,\dot{\vartheta} = \Omega\,F^*_{motion}$$

$$\bar{\Omega}\,\dot{\vartheta} = \bar{\Omega}F^*_{force}$$

Unified Motion & Force Control



Two decoupled Subsystems

$$\Omega\,\dot{\vartheta} = \Omega\,F^*_{motion}$$

$$\bar{\Omega}\,\dot{\vartheta} = \bar{\Omega}F^*_{force}$$

Humanoid Robotics	
Hualong Ren	humanoid dexterous hand systems
Rohan Maheshwari	navigation/path planning in dynamic environments; bipedal walking
Mishel Johns	Task planning; Predictive control algorithms
Chris Dembia	human motion synthesis
Medical Robotics	
Kyuwon Kim	exoskeleton or robotic limbs for patients
Shiquan Wang	Communication optimization for telemanipulator and the control under time delay
Haptics	
Chris Ploch	haptic teleoperation
Design	
Kevin Tong	Human Augmenting Exoskeleton Design for Ease of Use?!?!
Path/Trajectory Generation	
Minghan Shen	Robot Simultaneous Localization and Mapping (SLAM)
Laura Stelzner	path planning in dynamic environments
Paul Chen	
Thomas Lipp	Minimum time trajectories
Luke Allen	Path planning for walking robots
Matt Kiener	Task planning for compliant motions (contact situations)
Aerial robotics	
Martina Troesch	Flying robots without external motion capture or micro flying robots with flapping wings
Hao Jiang	Airplane Perching
Margaret Chapman	Perception and obstacle avoidance for aerial unmanned vehicles (UAVs)
Controller Design	
Brian Soe	Adaptive and learning control
Patrick Sherman	Nonlinear and adaptive control technipiques
Li Xuesen	Adaptive and learning control
Lipeng Alex Liang	Adaptive and Feedback control