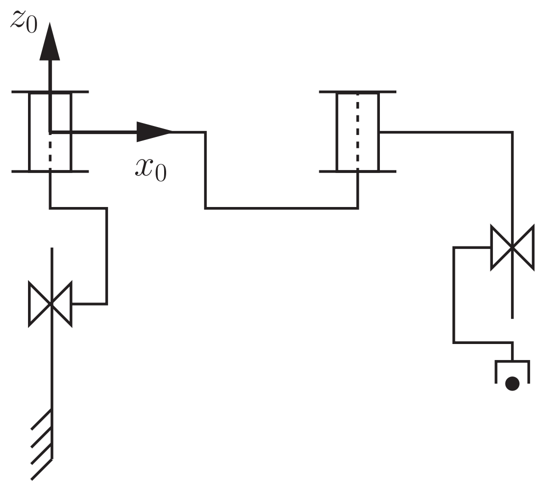
Advanced Robotic Manipulation Handout

CS327A (Spring 2015)

Problem Set #1 Due **Monday, Apr 13, 4pm**

1. [25 marks] **Jacobian Basics** : Consider the PRRP manipulator in the figure below.



For this manipulator, you are given the following homogeneous transformation matrices:

$$T_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{12} = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{23} = \begin{bmatrix} c_3 & -s_3 & 0 & l_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{34} = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Give a set of generalized coordinates for the manipulator.

(b) Give a set of operational coordinates for the end-effector.

(c) How many d.o.f. does the manipulator have? How many d.o.f. does the end-effector have? What is the degree of redundancy of the manipulator?

(d) If the manipulator is to position an object in space, what are your corresponding task configuration parameters? What is the degree of redundancy with respect to this task?

(e) If the manipulator is to orient an object, what are your corresponding task configuration parameters? What is the degree of redundancy with respect to this task?

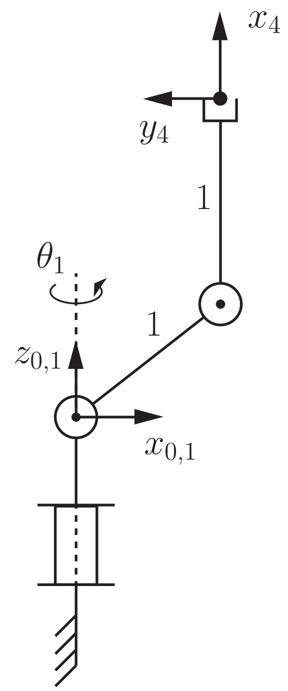
(f) Draw the frame {1} {2} {3} and {4}, then find the forward kinematics, T_{04}.

(g) Find the basic Jacobian matrix, $J_o(q)$.

(h) Find the Jacobian matrix, $J(q)$ for your operational space coordinate system choosen in (b).

(i) Find the corresponding E matrix such that $J(q) = E(X)J_o(q)$.

2. [30 marks] **Jacobian Without Rotation** : Consider the RRR-spatial manipulator in the figure below. For this manipulator, you are only to consider the *position* of the end-effector.



(a) Draw the frame $\{2\}$ and $\{3\}$, then find the forward kinematics, T_{04}.

(b) Find the Jacobian matrix in frame $\{0\}$.

(c) Find the Jacobian matrix in frame $\{2\}$.

(d) Find the singularities of the Jacobian.

(e) Draw the robot configurations at the sigularities found in (d). What are the singular directions? Explain the physical meaning.

(f) Now, let's consider the configuration when $\theta_2 = 90°$ and $\theta_3 = 0°$. Here, the manipulator can be treated as a redundant manipulator in the subspace orthogonal to the sigular directions.

   i. Find the Jacobian matrix in frame $\{2\}$. This matrix should be 1×3.

   ii. Find the pseudo inverse of the Jacobian matrix.

   iii. Find a matrix that will project a given vector to the associated null space.

   iv. Describe this configuration. Discuss your results with respect to the general solution:

$$\delta q = J^{+}\delta x + [I_3 - J^{+}J]\delta q_0.$$

   In particular, what are the resulting possible motions when you project an arbitrary vector δq_0 on to the null space?

3. [25 marks] **Jacobian Using Directional Cosines** : Consider the manipulator described by the following:

$$ {}^0_3T = \begin{bmatrix} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; {}^3_4T = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_4 & -C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; {}^4_5T = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^5_6T = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_6 & -C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } {}^0_6T = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & -C_4S_5 & d_1 \\ S_5C_6 & -S_5S_6 & C_5 & d_2 \\ -S_4C_5C_6 - C_4S_6 & S_4C_5S_6 - C_4C_6 & S_4S_5 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

(a) Draw a schematic of the robot, labeling all the generalized coordinates, and joint frames.

(b) Find the basic Jacobian matrix J_0.

(c) Determine the kinematic singularities and singular directions. Explain the physical meaning.

(d) Consider the representation $\mathbf{x} = [\mathbf{x}_p^T \; \mathbf{x}_r^T]^T$ consisting of the Cartesian coordinates for the position $\mathbf{x}_p$ of the wrist point and the direction cosines $\mathbf{x}_r$ for the orientation of the end-effector.

Find the Jacobian matrix J associated with this representation.

(e) Consider a starting configuration $\mathbf{q} = [1.0m \; 0.0m \; 0.5m \; 0° \; 90° \; 0°]^T$. and a desired end-effector goal configuration $\mathbf{x}_d = [\mathbf{x}_{pd}^T \; \mathbf{x}_{rd}^T]^T$ where $\mathbf{x}_{pd} = [0.9 \;\; 0.0 \;\; 0.6]^T$ and $\mathbf{x}_{rd} = [0.0 \;\; 1.0 \;\; 0.0 \;\; -0.12 \;\; 0.0 \;\; -0.99 \;\; -0.99 \;\; 0.0 \;\; 0.12]^T$.

Find the position error vector $\delta\mathbf{x}_p$ and the instantaneous angular error vector $\delta\Phi$.